PHYS 292 Notes

Eddie Guo
University of Alberta

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1 Dartboard Statistics

1.1 Statistics

1.1.1 Uncertainty Measurements

- Assertions of uncertainties w/o some explanation of how they were estimated are useless.
- If $t = 2.4$ s, report it as $t = 2.4 \pm 0.1$ s.
  - Uncertainties should almost always have 1 (or sometimes 2) significant figures.
  - Measured value should not be more accurate than the uncertainty (i.e., to same digit or decimal place as the standard error).
- Measurements *accurate* when *random* uncertainties reduced to an acceptable lvl.
- Measurements *precise* when *systematic* uncertainties reduced to an acceptable lvl.

1.1.2 Gaussian PDF

$$G(x, \bar{x}, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\bar{x})^2}{2\sigma^2}\right]$$

- Normalization: $\int_{-\infty}^{\infty} G(x, \bar{x}, \sigma)\ dx = 1$
- Mean: $\int_{-\infty}^{\infty} xG(x, \bar{x}, \sigma)\ dx = 1$
- Variance: $\int_{-\infty}^{\infty} (x-\bar{x})^2 G(x, \bar{x}, \sigma)\ dx = 1$

1.1.3 Other Impt Equations

$$\bar{x} = \mathbb{E}(X) = \frac{\sum x_i}{n}$$
$$\sigma_{N-1} = \sqrt{\frac{1}{N-1}\sum (x_i - \bar{x})^2}$$
$$\alpha = \frac{\sigma_{N-1}}{\sqrt{N}}$$

1.2 Lab

- Testing random dist'n of errors using darts.
- This lab is basically free marks so don't mess up lol.

2 Const-V Gas Thermometer

2.1 Statistics

2.1.1 Ordinary Least Squares (OLS)

$$y = A + Bx$$
$$P_{A,B}(y_i) \propto \frac{1}{\sigma_y} e^{(y_i A - Bx_i)^2/(2\sigma_y^2)}$$
$$P_{A,B}(y_i) = \prod_{i=1}^{N} P_{A,B}(y_i) \propto \frac{1}{\sigma_y^N} e^{-\chi^2/2}$$

where $\chi^2 = \sum_{i=1}^{N} \frac{(y_i - A - B_{x_i})^2}{\sigma_i^2}$. We can minimize χ^2 via partial derivatives wrt A and B to obtain:

$$\begin{cases} AN + B\sum x_i = \sum y_i \\ A\sum x_i + B\sum x_i^2 = \sum x_i y_i \end{cases}$$

And the solution to this system is:

$$A = \frac{(\sum x^2 \sum y - \sum x \sum xy)}{\Delta}$$
$$B = \frac{N\sum xy - \sum x \sum y}{\Delta}$$
$$\Delta = N\sum x^2 - (\sum x)^2$$

And the errors are:

$$\sigma_y = \sqrt{\frac{1}{N-2}\sum (y_i - A - Bx_i)^2}$$
$$\sigma_A = \sigma_y \sqrt{\frac{\sum x^2}{\Delta}}, \qquad \sigma_B = \sigma_y \sqrt{\frac{N}{\Delta}}$$

2.2 Lab

- Goals:
  - Det abs zero temp using ideal gas.
  - Measure P using MEMS (microelectromechanical system).
  - Perform lin reg using least squares.
  - Test if model (ideal gas) can explain expt.
- Procedure:

- Make P measruements of a const V.
- Anallyze data using ideal gas model.
- Obtain abs zero temp.

- $PV = nRT$
- Assume gas behaves like billiard balls w/ elastic collisions. Low density, no interactions outside collisions.
- Ctrl T and V to determine P.
- Gas trapped in bulb, heating water w/ bunsen burner. P measured w/ Raspberry Pi sensor (Honeywell TruStability® Board Mount Pressure Sensors). Works via piezoelectrics.

3 E/M Ratio

3.1 Statistics

If variables indep,

$$\sigma_f^2 = \sum_i \left(\frac{\partial f}{\partial x_i} \sigma_i \right)^2$$

If not indep, add covariance terms (linear approx):

$$\sigma_f^2 = \sum_i \left(\frac{\partial f}{\partial x_i} \sigma_i \right)^2 + \sum_{i \neq j} \sum \left(\frac{\partial f}{\partial x_i} \right) \left(\frac{\partial f}{\partial x_j} \right) \sigma_{ij}$$

3.2 Lab

- Goals
  - Bending of charges in **B** to est their E.
  - Analyze data from CRT.
  - Perform a linear fit, extract the e/m ratio from the slope of the fit.
- Use WLS (NOT OLS) for this experiment.
- R, v, and B relation: $mv = qBR$
- Energy relation: $E_k = \frac{1}{2} mv^2 = eV_A$
- Helmholtz coil B strength: $B_H = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 N I_H}{R}$
- The ratio e/m (or m/e) will be the slope of your fn.
- Vary electron kinetic energy and magnetic fields and record the bending radius of the cathode rays.
- Modify the settings to get the electrons to pass over each of the crossbars: which are placed at a known radius. You can:
  - Change V while keeping B constant,
  - Change B while keeping V constant, and
  - Change both B and V simultaneously.

4 Millikan's Oil Drop

4.1 Statistics

Consider the following cases for uncertainties in linear least squares fitting:

1. Error in y, no error in x: just use σ_y.
2. Error in x, no error in y: $\sigma_y(\text{equiv}) = \frac{dy}{dx} \sigma_x = B\sigma_x$
3. Error in x and y: $\sigma_y(\text{equiv}) = \sqrt{\sigma_y^2 + (B\sigma_x)^2}$

Remarks:

- σ_y and B computed using eqs in Section (2.1.1).
- Use dy/dx instead of B for non-linear least squares.
- Use a weighted fit if errors do not have the same uncertainty: $w_i = 1/\sigma_i^2$, where σ_i is the associated error for each y_i:

$$A = \frac{\sum wx^2 \sum wy - \sum wx \sum wxy}{\Delta}$$
$$B = \frac{\sum w \sum wxy - \sum wx \sum wy}{\Delta}$$
$$\Delta = \sum w \sum wx^2 - (\sum wx)^2$$
$$w_i = \frac{1}{\sigma_i^2}$$
$$\sigma_A = \sqrt{\frac{\sum wx^2}{\Delta}}, \qquad \sigma_B = \sqrt{\frac{\sum w}{\Delta}}$$

4.2 Lab

- Objectives:
  - Measure charge of electrons.
  - Experimentally est charge quantization.
- Experimental goals:
  - Use terminal velocities to est forces and charge.
  - Analyze data from droplets falling and rising uner diff forces.
  - Search for evidence of charge quantization.
- Procedure: track oil droplets in controlled setup. Measure their velocities, analyze the data.
- Millikan's expt showed:
  - Could suspend single oil drops using **E** to balance gravity.
  - Measured terminal velocity of oil drop to det its radius. Knowing the oil's density, we can find itss mass.
  - He found the charge each drop to be a multiple of $e = 1.601 \times 10^{-19}$ C.

- $a = 0 \Longrightarrow mg = kv_f$
- If each droplet has a charge, $qE = mg + kv_f$.
- Sphere moving in fluid: $kv_f = 6\pi\eta a^3\rho$, where k is friction ccoeff, η is viscosity, a is radius, ρ is density.

5 Superconductivity

5.1 Lab

- Goals
  - Establish whether a prepared material shows superconductivity.
  - Search for the Meissner effect.
  - Measre the resistance of a material.
- Meissner effect: expulsin of magnetic field from a superconductr when it's coooled below a critical tempterature, T_C.
  - Magnetic field lines "expelled" from an object when $T < T_C$ (i.e., no field lines pass through the object).
  - A ssupercurrent induced on surface to exactly cancel the external field within the material.
  - Requires extremely low temperatures.

6 Data Rejection

Chauvenet's criterion. (If taking more data impractical).

1. Assume all data points legitimate, take mean and standard deviation then quantify how anomalous the measurement is by finding its z-score.
2. Calc probability assoc w/ that z-score for both tails.
3. Determine number of measurements expected to be that extreme (multiply probability by number of data points collected).
4. Chauvenet says anything below 0.5 can be rejected.

Alternatively, obtain final results w/ and w/o rejection.

7 Franck-Hertz & Arbitrary Fits

7.1 Statistics

7.1.1 Fitting an Arbitrary Function

- In Franck-Hertz, valleys look like quadratic functions: $y = a(x+b)^2 + c$. We want to compute b.

```
import numpy as np
from scipy import optimize

def myquad(x, a, b, c):
    return a*(x-b)**2+c

# non-linear least squares
# popt = estimated parameters
# pcov = covariance matrix
popt, pcov = optimize.curve_fit(
    myquad, x, y,
    [100, 7, 50], # initial guess
    sigma=yerr)

perr = np.sqrt(np.diag(pcov))
```

7.2 Lab

- Defend atomic model w/ E lvls using Franck-Hertz expt (think Bohr and quantization).
- Goals:
  - Operate vacuum tube w/ hot cathode inside.
  - Measure small currents w/ an electrometer.
  - Est excitation E of Hg.
- Franck-Hertz
  1. Accelerate electrons to some known E (use parallel plate capacitor).
  2. Now place collector plate past the accelerating voltage. Put negative V_s on collector plate. Only e^- w/ $T > V_s$ will hit (stopping voltage).
  3. Continue increasing V_s until e^- start exciting Hg atoms and measure current.
  4. Once an Hg atom is excited, current drops suddenly then increases again (peaks and valleys in I vs V_s plot). Diff btw peaks and valleys is the E diff for raising Hg an energy state.

8 Radioactivity of Radon & Exponential Fits

8.1 Lab

- Understand and measure radioactive decays using radon.
- Goals:
  - Use leaf electroscope to collect charges.
  - Take differential measurement vs time.
  - Fit the half-life of radon-220.
- Radioactive decay relates to stability in binding the nucleus together.
- 3 types of decays:
  - α: Emission of an He nucleus (2p2n). Paper can stop it (thin).
  - β: Neutron to electron + neutrino. Thin sheet of metal can stop it (normal).
  - γ: Change in internal E config leading to emission of γ ray. Lead shielding can stop it (thick).
- Decay is a quantum mechanical effect.
- Decay rate: $\frac{dN_n}{dt} = -kN_n \implies N_n(t) = N_0 e^{-kt}$

  Half-life: $t_{1/2} = \ln 2/k, \quad k$ is prop const
- Th-232 has a long decay series.

8.2 Statistics

- Linear fit: simple, can be done by hand.
- Arbitrary curve fit: requires a computer.
- Exponential functions can be linearized.

  $N_n = N_0 e^{-kt} \implies \ln N_n = \ln N_0 - kt$

9 Faraday Effect & Goodness of Fit

9.1 Statistics

- Objectives: Corroborate Malus' law and explore Faraday rotation.
- Goals:
  - Explore polarization of light.
  - Learn how to measure small signals.
- Procedure
  - Measure intensity of polarized beam.
  - Pass beam thru dielectric material.
  - Pass beam thru material in magnetic field.
- 3 types of polarized light: linear (plane waves), circular (90° phase diff), elliptical (2 plane waves of diff amplitude but relative phase shift of 90°).
  - Linear pol: $\mathbf{E}_l = E_0 \cos(kz - \omega t + \phi_1)\hat{x}$

- Circular pol: $\mathbf{E}_c = E_0\cos(kz-\omega t+\phi_2)\hat{x} - E_0\sin(kz-\omega t+\phi_2)\hat{y}$
- Elliptical pol: $E_l(x,y,0)\hat{x} + E_l(x,y,\frac{-\pi}{2})\hat{y}$

- Can make linearly polarized light by expressing it as circularly polarized beams: $\mathbf{E}_l = \mathbf{E}_c(x,y,\phi) + \mathbf{E}_c(x,y,-\phi)$
- Light interacts w/ materials (light can't interact w/ light due to superposition), so can polarize beam of unpolarized light by decoupling their components.
- $\Delta\theta = VBL$, V is Verdet's constant (how strongly B affects refractive index), L is length of medium.
- Malus' law: $I = I_0\cos^2\theta$, since $V \propto I \propto E^2$ then $V = V_0\cos^2(\theta-\theta_0)$

9.2 Lab

Recall: $\chi^2 = \sum_{i=1}^{N}\frac{(y_i-A-Bx_i)^2}{\sigma_i^2}$. Let's generalize this:

$$\chi^2 = \sum_{i=1}^{N}\frac{(y_i-y_{m,i})^2}{\sigma_i^2}$$

where $y_{m,i}$ is the model or expected value. This parameter can be used to test the agreement btw a dataset and a model.

Degrees of freedom: $\nu = N_{\text{data}} - N_{\text{params}}$. Goodness of fit can be estimated using the reduced chi-squared:

$$\chi^2_\nu = \left(\sum_{i=1}^{N}\frac{(y_i-y_{m,i})^2}{\sigma_i^2}\right)/\nu \tag{1}$$

Naively, expect each dof contributes 1 unit, dist'n.

- $\chi^2_\nu = 0 \implies$ no uncertainty and perfect fit w/ dist'n.
- $\chi^2_\nu = 1 \implies$ data well described by proposed dist'n.
- $\chi^2_\nu \gg 1 \implies$ unlikely data comes from proposed dist'n.

Can compute p-value via integration of pdf of the chi-squared dist'n w/ a given dof.

```
import numpy as np
from scipy import stats

# assume variables exist
chi_sq = np.sum((y_exp - y_model)**2 / error**2)
dof = x.length() - 1
reduced_chi_sq = chi_sq / dof
p = 1 - stats.chi2.cdf(chi_sq, dof)
print(f'p-value: {p}')
```

10 Covariance & Correlation

10.1 Derivation of Covariance

$$\begin{aligned}
q_i &= q(x_i,y_i)\\
q_i &\approx q(\bar{x},\bar{y}) + \frac{\partial q}{\partial x}(x_i-\bar{x}) + \frac{\partial q}{\partial y}(y_i-\bar{y})\\
\bar{q} &= \frac{\sum_i q_i}{N} = q(\bar{x},\bar{y})\\
&= \frac{1}{N}\sum_i\left(q(\bar{x},\bar{y}) + \frac{\partial q}{\partial x}(x_i-\bar{x}) + \frac{\partial q}{\partial y}(y_i-\bar{y})\right)\\
\sigma_q^2 &= \frac{1}{N}\sum_i(q_i-\bar{q})^2\\
\sigma_q^2 &= \left(\frac{\partial q}{\partial x}\right)^2\underbrace{\frac{1}{N}\sum_i(x_i-\bar{x})^2}_{\sigma_x^2} + \left(\frac{\partial q}{\partial y}\right)^2\underbrace{\frac{1}{N}\sum_i(y_i-\bar{y})^2}_{\sigma_y^2}\\
&\quad + 2\frac{\partial q}{\partial x}\frac{\partial q}{\partial y}\underbrace{\frac{1}{N}\sum_i(x_i-\bar{x})(y_i-\bar{y})}_{\sigma_{xy}}\\
&= \left(\frac{\partial q}{\partial x}\right)^2\sigma_x^2 + \left(\frac{\partial q}{\partial y}\right)^2\sigma_y^2 + 2\frac{\partial q}{\partial x}\frac{\partial q}{\partial y}\sigma_{xy}
\end{aligned}$$

10.2 Remarks

- Covariance is $\sigma_{xy} = \frac{1}{N}\sum_i(x_i-\bar{x}_i)(y_i-\bar{y})$.
- If $\sigma_{xy}\neq 0$, errors of x and y are correlated.
- Schwarz's inequality: $|\sigma_{xy}| \leq \sigma_x\sigma_y \implies \sigma_q \leq \left|\frac{\delta q}{\delta x}\right|\sigma_x + \left|\frac{\delta q}{\delta y}\right|\sigma_y$
- If you have more variables, σ_{xy} becomes a matrix.

10.3 Correlation

$$r = \frac{\sigma_{xy}}{\sigma_x\sigma_y} \tag{2}$$