PHYS 130 Formula Sheet

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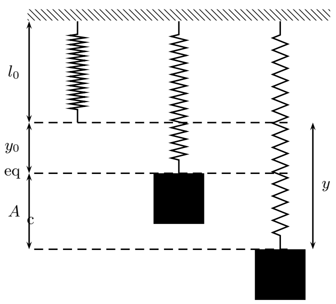
SI Prefixes

Deca (da, 10^1), hecto (h, 10^2), kilo (k, 10^3), mega (M, 10^6), giga (G, 10^9), tera (T, 10^{12}), peta (P, 10^{15}), exa (E, 10^{18}), zetta (Z, 10^{21}), yotta (Y, 10^{24})

Deci (d, 10^{-1}), centi (c, 10^{-2}), milli (m, 10^{-6}), micro (μ, 10^{-6}), nano (n, 10^{-9}), pico (p, 10^{-12}), femto (f, 10^{-15}), atto (a, 10^{-18}), zepto (z, 10^{-21}), yocto (y, 10^{-24})

Simple Harmonic Motion



$\omega = 2\pi f = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$

$\mathbf{F} = m\ddot{x} = -kx$

$k = m\omega^2$ $\qquad$ $\ddot{x} = -\omega^2 x$

$k_{\text{eff}} = \sum_{i=1}^{n} k_i$ (parallel)

$k_{\text{eff}} = \left(\sum_{i=1}^{n} \frac{1}{k_i}\right)^{-1}$ (series)

$x(t) = A\cos(\omega t + \phi)$

$v(t) = -A\omega \sin(\omega t + \phi)$, $\frac{\pi}{2}$ ahead of x

$a(t) = -A\omega^2 \cos(\omega t + \phi)$, π ahead of x

$v_{\max} = A\omega$ (at eq.) $\qquad$ $a_{\max} = A\omega^2$ (at $A_{\max}$)

Energy and Initial Conditions

$v = \omega\sqrt{A^2 - x^2}$ $\qquad$ $A = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}}$

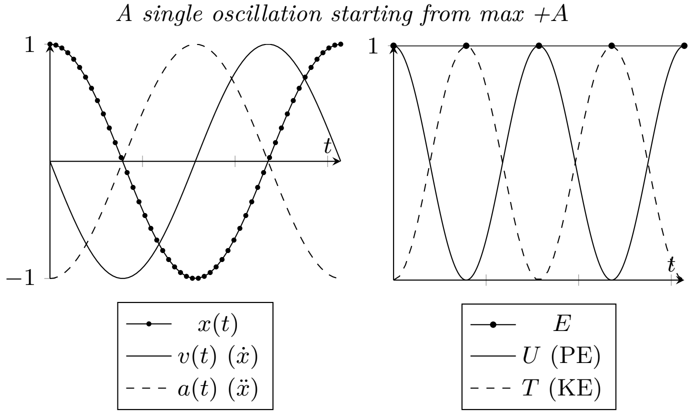
$\phi = \tan^{-1}\left(-\frac{v_0}{\omega x_0}\right) \Rightarrow$ check sign of x and $\dot{x} \Rightarrow \phi \pm \pi$ on $(-\pi, \pi)$

If you take $\cos^{-1} x$, then check $\pm\theta$

$E = T + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$

$T = \frac{1}{2}k(A-x)^2$ $\qquad$ $U = \frac{1}{2}kx^2$ $\qquad$ $T = U$ at $\frac{A}{\sqrt{2}}$ every $\frac{\pi}{2\omega} = \frac{T}{4}$

A single oscillation starting from max +A



Small Angle Pendulums

$\ddot{\theta} = -\frac{g}{l}\theta$ $\qquad$ $\omega = 2\pi f = \frac{2\pi}{T} = \sqrt{\frac{g}{l}}$

$\theta = \frac{s}{l} = \omega t + \phi$ $\qquad$ s = arc length, l = length

$s = l\theta = l\left[\theta_0 \cos(\omega t + \phi)\right]$ $\qquad$ $v_{\max} = \omega l \theta_0$ $\qquad$ $a_{\max} = \omega^2 l \theta_0$

Damped Oscillators

$\ddot{x} + 2\zeta\omega_0\dot{x} + \omega_0^2 x = 0$ $\qquad$ $\omega_0 = \sqrt{\frac{k}{m}}$ $\qquad$ $\zeta = \frac{b}{2\sqrt{mk}} = \frac{b}{2m\omega_0}$

$q = \omega_0\left(-\zeta \pm \sqrt{\zeta^2 - 1}\right)$

Overdamped ($\zeta > 1$, $b^2 > 4mk$)

$$x(t) = e^{-\omega_0\zeta t}\left(Ae^{\omega_0 t\sqrt{\zeta^2-1}} + Be^{-\omega_0 t\sqrt{\zeta^2-1}}\right)$$

Critically Damped ($\zeta = 1$, $b^2 = 4mk$)

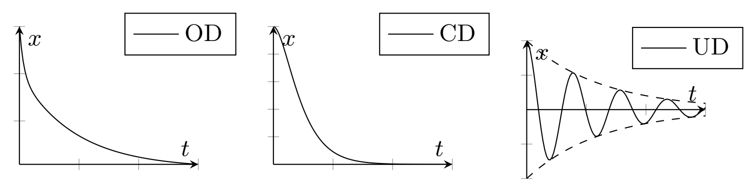
$$x(t) = (A + Bt)e^{-\omega_0 t}$$

$$\dot{x} = -A\omega_0 e^{-\omega_0 t} + Be^{-\omega_0 t} - B\omega_0 t e^{-\omega_0 t}$$

Underdamped ($\zeta < 1$, $b^2 < 4mk$)

$$x(t) = \underbrace{A\,\overbrace{e^{-\omega_0\zeta t}}^{\text{decay envelope}}}_{\text{amplitude}} \cos\underbrace{(\overbrace{\omega_0\sqrt{1-\zeta^2}}^{\text{angular freq }(\omega_{\text{damped}})}\,t + \phi)}_{\text{phase}}$$

$A = A_0 e^{-\omega_0\zeta t}$ $\qquad$ $\omega_{\text{damped}} = \omega_0\sqrt{1-\zeta^2}$



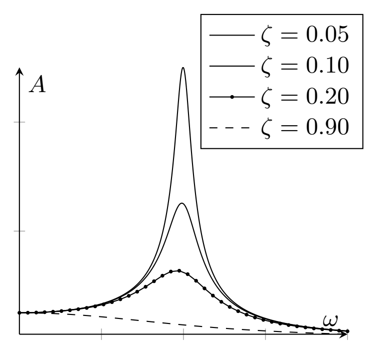
Driven Oscillators and Resonance

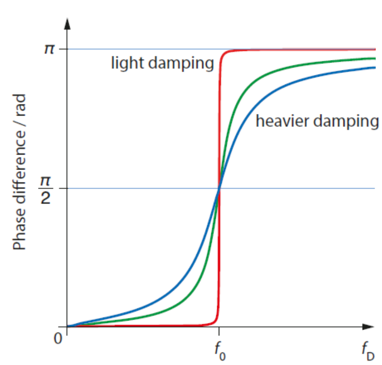
$m\ddot{x} = -kx - b\dot{x} + F_0\cos(\omega t) \quad \Rightarrow \quad \ddot{x} + 2\zeta\omega_0\dot{x} + \omega_0^2 x = \frac{F_0}{m}\cos(\omega t)$

$A = \frac{F_0}{m}\frac{1}{\sqrt{(\omega_0^2-\omega^2)^2 + 4\omega_0^2\omega^2\zeta^2}}$ $\qquad$ $\phi = \tan^{-1}\left(\frac{2\omega_0\omega\zeta}{\omega_0^2 - \omega^2}\right)$

$\omega_r = \omega_0\sqrt{1 - 2\zeta^2}$ $\qquad$ $\zeta < \frac{1}{\sqrt{2}}$ $\qquad$ $\zeta \ll 1, \omega_r \approx \omega_0$

$\lim_{\omega\to 0}\phi = 0$ (low ω) $\qquad$ $\lim_{\omega\to\omega_0}\phi = \pi/2$ ($\omega = \omega_0$) $\qquad$ $\lim_{\omega\to\infty}\phi = \pi$ (high ω)





$E = \frac{1}{2}kA^2 e^{-2\omega_0\zeta t} = \frac{1}{2}kA^2 e^{-bt/m}$ $\qquad$ $\omega = \omega_0 \approx \omega_r$ at $\Delta\phi = \frac{\pi}{2}$

Waves

Wave number: $k = \frac{2\pi}{\lambda}$

Phase velocity: $c = f\lambda = \frac{\omega}{k}$ $\left(t = \frac{\lambda}{c} = \frac{1}{f}\right)$

Wave on string: $c = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{TL}{M}}$ $\mu = \frac{M}{L}$ (linear mass density)

Wave equation: $\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2}$

Transverse position: $\psi_{\pm}(x,t) = A\cos(kx \mp \omega t + \phi)$

Bulk modulus (Pa, N/m^2): $B = -\frac{\Delta P}{\Delta V/V_0}$ $\rho = \frac{m}{V}$ $P = \frac{F}{A}$

Lower $B \to$ higher compressibility (B usually +ve)

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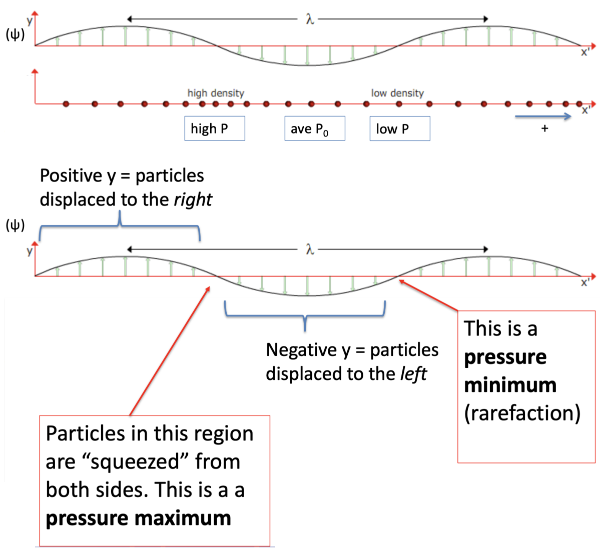
Acoustic Waves

$\frac{\partial^2 P}{\partial x^2} = \frac{\rho}{B}\frac{\partial^2 P}{\partial t^2}$ $p(x,t) = -B\frac{\partial \psi}{\partial x}$ $c = \sqrt{\frac{B}{\rho}} = \frac{\omega}{k} = f\lambda$

$p(x,t) = BkA_\psi \sin(kx - \omega t + \phi)$

$Bk = \frac{\omega^2 \rho}{k} = c\omega\rho \Rightarrow BkA_\psi = c\rho\omega A_\psi$ A_ψ is displacement A

Acoustic amplitude: $A_p = BkA_\psi = c\rho\omega A_\psi$ $\Rightarrow$ $A_\psi = \frac{A_p}{Bk} = \frac{A_p}{c\rho\omega}$



y vs $x \to$ snapshot y vs $t \to$ position of 1 particle into future

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Wave Power and Intensity

Units: $[P] = \text{W}$, $[I] = \text{W/m}^2$, $[\beta] = \text{dBSIL}$

$P = T_y v = T\omega k A^2 \sin^2(kx - \omega t + \phi) = \sqrt{T\mu}\,\omega^2 A^2 \sin^2(kx - \omega t + \phi)$

Wave on string (one direction): $\langle P \rangle = \frac{1}{2}\sqrt{T\mu}\,\omega^2 A^2$

Wave on string (both directions): $\langle P \rangle = \sqrt{T\mu}\,\omega^2 A^2$

Acoustic wave: $\langle I \rangle = \frac{1}{2}B\omega k A^2 = \frac{1}{2}\sqrt{B\rho}\,\omega^2 A^2$

$\beta = 10\log\left(\frac{I}{I_0}\right)$ $I_0 = 10^{-12}$ W/m^2 (threshold of audibility)

$\beta_2 - \beta_1 = 10\log\left(\frac{I_2}{I_1}\right)$ $I = \frac{P}{A}$ (think surface area)

Intensity at a Wave Front

Sphere

- $I = \frac{P}{4\pi r^2}$
- $I \propto P \propto A^2 \propto \omega^2 \propto \frac{1}{r^2}$

Circle

- $I = \frac{P}{2\pi r}$
- $I \propto P \propto A^2 \propto \omega^2 \propto \frac{1}{r}$

. . . .

Superposition and Interference

$\Delta\phi = 0$, same $f \to$ add A $\Delta\phi \neq 0$, diff $f \to$ add I

Identical λs in same dir w/ constant $\Delta\phi$:

$$\psi_{\text{net}}(x,t) = \underbrace{2y_m \cos\left(\frac{\phi}{2}\right)}_{A \text{ of combined } \lambda\text{s}} \sin\left(kx - \omega t + \frac{\phi}{2}\right)$$

Fully constructive when: $\sin\theta = \frac{n\lambda}{d}$

- ϕ is even multiple of π (0, 2π, 4π, ...)
- $\Delta L = n\lambda$ (whole multiples of λ)

Fully destructive when: $\sin\theta = \frac{(n+\frac{1}{2})\lambda}{d}$

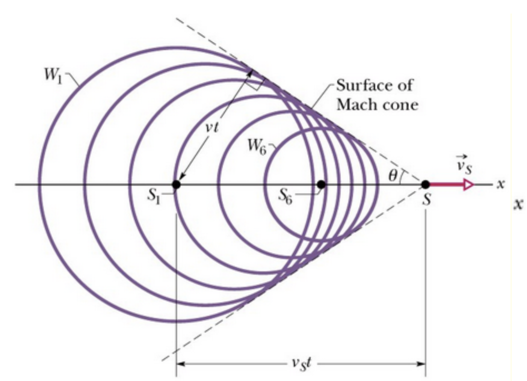
- ϕ is odd multiple of π (π, 3π, 5π, ...)
- $\Delta L = \left(n + \frac{1}{2}\right)\lambda$ (odd $\lambda/2$)

$\frac{\Delta L}{\lambda} = \frac{\phi}{2\pi}$ $\Rightarrow$ $\phi = k\Delta L$ $\Delta L = d\sin\theta$

Beats (in-phase): $\psi_{\text{net}} = A\cos(\omega_1 t) + A\cos(\omega_2 t)$

$$\psi_{\text{net}} = \underbrace{2A\cos\left(\frac{\omega_1 - \omega_2}{2}t\right)}_{\text{time-varying } A \text{ (beats)}} \cos\left(\underbrace{\frac{\omega_1 + \omega_2}{2}}_{\text{pitch}}t\right)$$

Beat freq ($\Delta\omega$): $f_b = |f_1 - f_2|$ Apparent freq ($\bar{\omega}$): $f' = \frac{f_1 + f_2}{2}$



Mach cone: $\sin\theta = \frac{c}{v_s} = \frac{v_P}{v_s} = \frac{v \text{ of } \lambda}{v \text{ of src}}$ Mach number: $\frac{v_s}{c}$

. . . .

Trigonometry

Basic Identities

- $\sin x = \sin(\pi - x)$ arcsin $\Rightarrow x_2 = \pi - x_1$
- $\cos x = \cos(-x)$ arccos $\Rightarrow x_2 = -x_1$
- $\tan x = \tan(\pi + x)$ arctan $\Rightarrow x_2 = \pi + x_1$
- $\cos(\theta \pm \pi) = -\cos(\theta)$
- $\cos\theta = \sin\left(\theta + \frac{\pi}{2}\right)$
- $\sin\theta = \cos\left(\theta - \frac{\pi}{2}\right)$

Addition/Subtraction Identities

- $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$
- $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
- $\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$
- $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

Doppler $f_0 = f_s \frac{c\pm v_0}{c\mp v_s}$

Observer

- $+v_0$ = towards source
- $-v_0$ = away from source

Source

- $-v_s$ = towards observer
- $+v_s$ = away from observer

Standing Waves

General form: $\psi(x,t) = 2A\sin(kx)\sin(\omega t)$

Both ends open/closed: $\lambda = \frac{2l}{n}$, $f = \frac{nc}{2l}$

1 open, 1 closed: $\lambda = \frac{4l}{2n-1}$, $f = \frac{(2n-1)c}{4l}$

Closed Boundary (inverted)

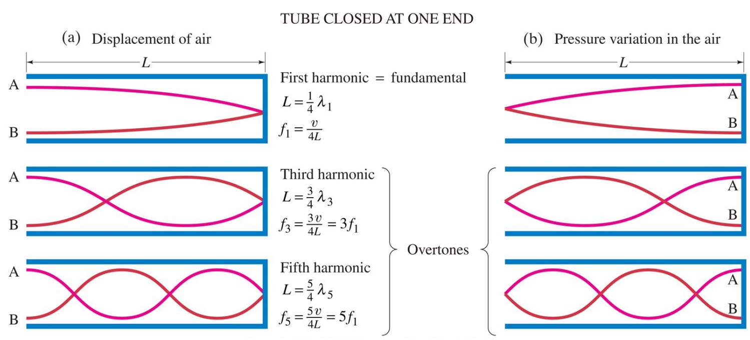
$\psi_i(x_0) + \psi_r(x_0) = 0 \qquad \Delta\phi = \pi$

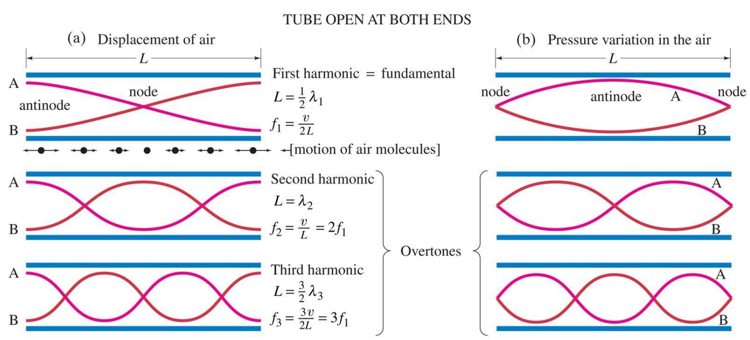
Open end → anti-node (max A)

Open Boundary (not inverted)

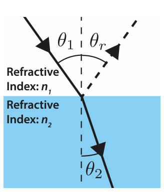
$\frac{\partial\psi}{\partial x} = 0 \qquad \Delta\phi = 0$

Closed end → node ($A = 0$)





Geometric Optics



Reflection: $\theta_i = \theta_r$

Snell's law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\sin\theta_c = \frac{n_2}{n_1}$, $n_1 > n_2$ $\qquad \theta_c < \theta$ is TIL

$n = \frac{c}{v_p} = \frac{v \text{ in vacuum}}{v \text{ in medium}}$, $n \geq 1$

Total trapping of light ocurs when material surrounded by lower n

Thin lens: $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ $\qquad$ Linear magnification: $M = \frac{I}{O} = -\frac{v}{u}$

Sign Conventions

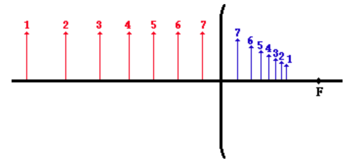
$f < 0$ for diverging mirrors (convex) and lenses (concave)

$v > 0 \to$ real $\qquad v < 0 \to$ virtual

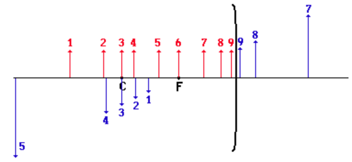
$I,\ M > 0 \to$ upright $\qquad I,\ M < 0 \to$ inverted

Mirrors: img opposite to obj → virtual

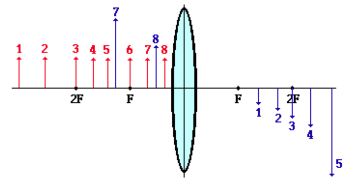
Lenses: diverging rays → virtual



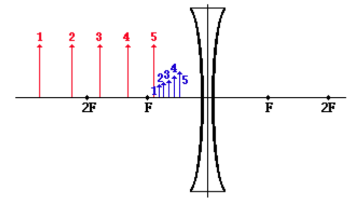
(a) Convex mirror (div)



(b) Concave mirror (conv)



(c) Convex lens (conv)



(d) Concave lens (div)

Optical Instruments

Lensmaker's equation: $\frac{1}{f} = \left(\frac{n}{n_o} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \qquad n_o \approx 1$ in air

Combined f (touching): $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

Combined f (sep by d): $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$

$R_i > 0$ if convex, $R_i < 0$ if concave

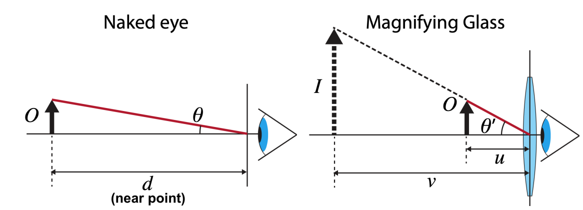


Figure 2: Magnifying glass: $M_{\max} = 1 + \frac{d}{f} \qquad M_{\min} = \frac{d}{f}$

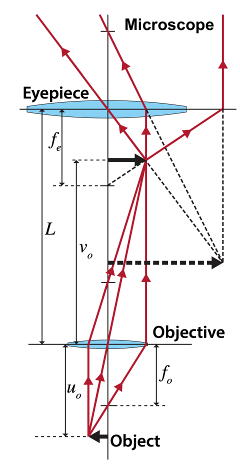


Figure 3: Microscope: object placed just over 1 f_o away. Intermediate image formed w/in 1 f_e. $\quad M = m_o m_e = -\frac{L}{f_0}\left(1 + \frac{d}{f_e}\right)$

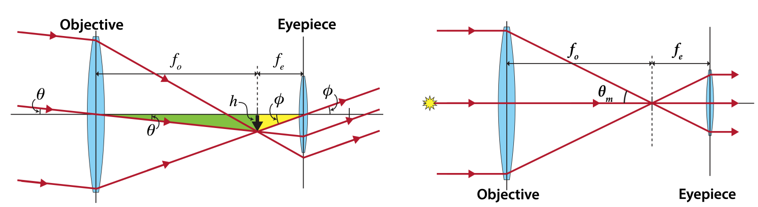


Figure 4: Telescope: $M_\theta = \frac{\phi}{\theta} = -\frac{f_0}{f_e} (= -\frac{d_0}{d_e}$ if all light captured). Here, d_0 = objective diam, d_e = eyepiece diam, $f_0 + f_e = L$. L is distance btw eyepiece 1° mirror/obj lens).

Polarization

If I_0 unpolarized: $I = \frac{1}{2} I_0$ Else: $A = A_0 \cos\phi$ $I = I_0 \cos^2\phi$

Brewster's angle: $\tan\theta_B = \frac{n_2}{n_1}$ ($\theta_B = \theta_i = \theta_r$; light hits n_2)

- *Reflected* ray *completely* polarized
- *Refracted* beam is *partially* polarized

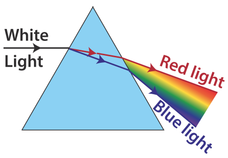
Dispersion

$\psi(x,t) = 2A\cos\left(\frac{1}{2}\Delta kx - \frac{1}{2}\Delta\omega t\right)\cos(\bar{k}x - \bar{\omega}t)$

$\bar{k} = \frac{1}{2}(k_1 + k_2)$ $\bar{\omega} = \frac{1}{2}(\omega_1 + \omega_2)$ $\Delta k = k_1 - k_2$ $\Delta\omega = \omega_1 - \omega_2$

Phase velocity: $v_p = \frac{\omega}{k}$ Group velocity: $v_g = \frac{d\omega}{dk}$

Normal dispersion: n greater for shorter λ



$v_p < v_g \Rightarrow$ anomalous dispersion

$v_p = v_g \Rightarrow$ no dispersion

$v_p > v_g \Rightarrow$ normal dispersion

Interference

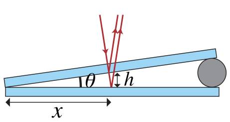
The type of interference only det by reflection $\Delta\phi$

$\Delta L = 2h = \frac{m\lambda}{n}$

Thin film formulae for $n = 1, 2, 3, \ldots$ $n_1\lambda_1 = n_2\lambda_2$

REMEMBER TO ACCOUNT FOR n!!!

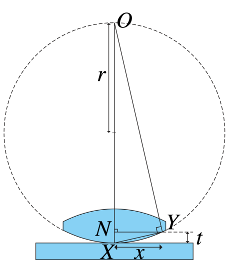
	Constructive	Destructive
No/both phase change	$h = n\lambda/2$	$h = (2n-1)\lambda/4$
One phase change	$h = (2n-1)\lambda/4$	$h = n\lambda/2$



Thin wedge (for $n = 0, 1, 2 \ldots$)

Constructive: $x = \frac{(2n+1)\lambda}{4\tan\theta}$

Destructive: $x = \frac{n\lambda}{2\tan\theta}$



Newton's rings (for $n = 0, 1, 2, \ldots$)

$t \approx \frac{x^2}{2r}$

Constructive: $x = \sqrt{\left(n + \frac{1}{2}\right)\lambda r}$

Destructive: $x = \sqrt{n\lambda r}$

Middle dark w/ decreasing width (due to $\sqrt{n}$)

Interferometers

$t_{||} - t_{\perp} \approx \frac{Lv^2}{c^3}$ where v is speed of "aether"

- $\Delta L = 2\Delta x = \frac{m\lambda_0}{2}$ ($\Delta\phi = \pi$; pattern inversion)
- $\Delta L = 2\Delta x = m\lambda_0$ ($\Delta\phi = 2\pi$; 1 fringe shift)
- ΔL is path length diff and Δx is mirror mvmt

Diffraction (General)

Fringe width $\propto \lambda \propto 1/d \propto 1/a \propto 1/I$

$N = \frac{L}{\lambda} = \frac{2x}{\lambda}$ N is λs that fits in length L

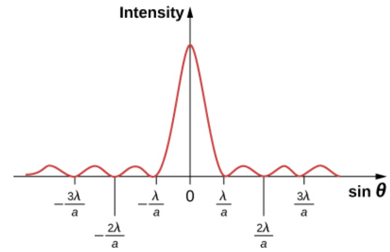
$m = N_2 - N_1 = \frac{L}{\lambda}(n_2 - n_1)$ n is refractive index ($L = 2x$)

$\text{sinc}\, x = \frac{\sin x}{x}$ **USE RADIANS!!!**

Single Slit Diffraction

$\psi(\theta) = A\cos(kr - \omega t)\ \text{sinc}\left(\frac{\pi}{\lambda} a \sin\theta\right)$ $I = I_0\ \text{sinc}^2\left(\frac{\pi}{\lambda} a\sin\theta\right)$

At min: $\Delta L = a\sin\theta = n\lambda$ $n = \pm1, \pm2, \ldots$ (NOT 0)



Double Slit Diffraction

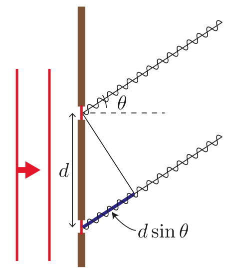
Narrow Slit

$\psi(\theta) = 2A\cos(\frac{\pi d}{\lambda}\sin\theta)\cos(kr - \omega t)$

$I = I_0\cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right)$

For $m = 0, \pm1, \pm2, \ldots$

- Max I: $d\sin\theta = m\lambda$
- Min I: $d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$



$x = \frac{L}{d}(n_2 w_2 - n_1 w_1)$

- x is shift of central max L is distance to screen
- n is refractive index w is width of slit covering

Wide Slit

- Double narrow slit fn multiplied by single slit fn
- a is slit width, d is distance btw slits

$m = \frac{d}{a}$ where m is the missing order

$\psi(\theta) = 2\psi_1\cos\left(\frac{\pi d}{\lambda}\sin\theta\right)$ where $\psi_1 = A\cos(kr - \omega t)\ \text{sinc}\left(\frac{\pi}{\lambda} a\sin\theta\right)$

$I = 4I_1\cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right)$ where $I_1 = I_0\ \text{sinc}^2\left(\frac{\pi}{\lambda} a\sin\theta\right)$

Diffraction Grating

Bright lines: $\sin\theta = \frac{m\lambda}{d} = mN\lambda$ $m = 0, 1, 2, \ldots$ and $N = 1/d$

$\Delta\lambda = \frac{\lambda}{Nm}$

d is spacing btw 2 lines N is number of slits per L

Rayleigh Criterion (Circular Aperture)

$\theta_R = \frac{1.22\lambda}{a} = \frac{D}{L}$ (for small θ to resolve 2 srcs)

- a is diameter of telescope
- D is distance btw 2 barely resolvable points
- L is distance to obj

Resolving power: $R = \frac{\lambda_{\text{avg}}}{\delta\lambda} \leq mNw$ (lines in grating $= Nw$)

$\sin\theta \approx \theta = mN\lambda = \frac{\lambda}{w}$ $\delta\theta = mN\delta\lambda \geq \frac{\lambda}{w} \Rightarrow R = \frac{\lambda}{\delta\lambda} \leq mNw$