

Strategy for Series

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Test for Divergence

Is $\lim_{n \rightarrow \infty} a_n = 0$?

NO

$\sum a_n$ diverges

YES

p-Series

Is $a_n = 1/n^p$?

YES

Is $p > 1$?

YES

$\sum a_n$ converges

NO

$\sum a_n$ diverges

NO

Geometric Series

Is $a_n = ar^{n-1}$ for $n \geq 1$?

YES

Is $|r| < 1$?

YES

$\sum_{n=1}^{\infty} a_n = \frac{a}{1-r}$

NO

$\sum a_n$ diverges

NO

Telescoping Series

Do subsequent terms cancel? May need to use partial fractions, logarithms, etc.

YES

Is $\lim_{n \rightarrow \infty} S_n = L \neq \pm\infty$?

YES

$\sum a_n$ converges

NO

$\sum a_n$ diverges

NO

Alternating Series

Is $a_n = (-1)^n b_n$ where $b_n \geq 0$?

YES

Is $b_{n+1} \leq b_n$ & $\lim_{n \rightarrow \infty} b_n = 0$?

YES

$\sum a_n$ converges

NO

Taylor Series

Is $a_n = \frac{f^{(n)}(a)}{n!} (x-a)^n$?

YES

Is $|x| < R$?

YES

$f(x) = \sum_{n=0}^{\infty} a_n$

NO

$\sum a_n$ diverges

NO

Try one or more of the following tests:

Comparison Test

Pick b_n . Does $\sum b_n$ converge?

YES

Is $a_n \leq b_n$?

YES

$\sum a_n$ converges

NO

NO

Is $a_n \geq b_n$?

YES

$\sum a_n$ diverges

Limit Comparison Test

Pick b_n . Is $\lim_{n \rightarrow \infty} a_n/b_n = L$, $L > 0 \neq \pm\infty$, and $a_n, b_n > 0$?

YES

Does $\sum_{n=1}^{\infty} b_n$ converge?

YES

$\sum a_n$ converges

NO

$\sum a_n$ diverges

Integral Test

Let $f(x) = a_x$; is $f(x)$ continuous, positive, and decreasing on $[a, \infty)$?

YES

Does $\int_a^{\infty} f(x) dx$ converge?

YES

$\sum a_n$ converges

NO

$\sum a_n$ diverges

Ratio Test

Is $\lim_{n \rightarrow \infty} |a_{n+1}/a_n| \neq 1$?

YES

Is $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| < 1$?

YES

$\sum a_n$ abs conv

NO

$\sum a_n$ diverges

Root Test

Is $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} \neq 1$?

YES

Is $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} < 1$?

YES

$\sum a_n$ abs conv

NO

$\sum a_n$ diverges