

# MAT E 201 Final Summary

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## 1 Composite Materials

**Definition 1.1.** (Composite materials). System of  $\geq 2$  components with an interface separating them.

Particulate composites	Contains large amounts of coarse particles. Designed to produce unusual properties rather than to increase strength.
Fiber composites	Improved strength, fatigue resistance. Strength may be high at room temp and elevated temps.
Laminar composites	Thin coatings designed to improve corrosion resistance while being low cost, high strength, or lightweight. Abrasion resistance, improved appearance, unusual thermal expansion characteristics.

**Formula 1.2.** (Rule of mixtures). For a composite with density  $\rho_c$ , with the  $i$ th component with a fraction  $f_i$  and density  $\rho_i$ ,

$$\rho_c = \sum_{i=1}^n f_i \rho_i \quad \text{and} \quad \sum_{i=1}^n f_i = \frac{\sum_{i=1}^n \rho_i V_i}{\rho_c V_c} = 1 \quad (1)$$

Similarly for thermal conductivity  $k$  and electric conductivity  $\sigma$ ,

$$k_c = \sum_{i=1}^n f_i k_i \quad \text{and} \quad \sigma_c = \sum_{i=1}^n f_i \sigma_i \quad (2)$$

Volume fraction for a component  $B$  can be computed using this formula:

$$f_B = \frac{w_B / \rho_B}{w_B / \rho_B + w_A / \rho_A} \quad (3)$$

**Example 1.3.** Spherical silica particles (100 nm in diameter) are added to vulcanized rubber in tires to improve stiffness. If the density of the vulcanized rubber matrix is  $1.1 \text{ g/cm}^3$ , the density of silica is  $2.5 \text{ g/cm}^3$ , and the tire has a porosity of 4.5%, calculate the number of silica particles lost when a tire wears down 0.4 cm in thickness. The density of the tire is  $1.2 \text{ g/cm}^3$ ; the overall tire diameter is 63 cm; and it is 10 cm wide.

*Solution.* We first compute the fraction of silica in the tire.

$$\begin{aligned}
1 &= f_{\text{rubber}} + f_{\text{silica}} + f_{\text{pores}} \implies f_{\text{rubber}} = 1 - 0.045 - f_{\text{silica}} = 0.955 - f_{\text{silica}} \\
1.2 \text{ g/cm}^3 &= (0.955 - f_{\text{silica}})(1.1 \text{ g/cm}^3) + f_{\text{silica}}(2.5 \text{ g/cm}^3) \implies f_{\text{silica}} = 0.107 \\
\Delta V &= \frac{\pi}{4} w (d^2 - (d - 2w)^2) = \frac{\pi}{4} (10)(63^2 - (63 - 2 \times 0.4)^2) = 787 \text{ g/cm}^3 \\
\Delta V_{\text{silica}} &= f_{\text{silica}} \Delta V = 0.107 \times 787 = 84.17 \text{ cm}^3 \\
N_{\text{silica}} &= \frac{\Delta V_{\text{silica}}}{V_{\text{silica particle}}} = \frac{84.17}{\frac{4}{3}\pi(100 \times 10^{-9}/2)^3} = 1.61 \times 10^{23} \text{ silica particles lost}
\end{aligned}$$

**Example 1.4.** A copper-silver bimetallic wire 1 cm in diameter is prepared by co-extrusion with copper as the core and silver as the outer layer. The desired properties along the length of the bimetallic wire are

- Thermal conductivity  $> 410 \text{ W / (m-K)}$ ,
- Electric conductivity  $> 60 \times 10^{-6} \Omega\cdot\text{m}$ ,
- Weight  $W < 750 \text{ g/m}$ .

	Copper	Silver
Density ( $\text{g/cm}^3$ )	8.96	10.49
$\sigma$ ( $\Omega^{-1} \cdot \text{m}^{-1}$ )	$59 \times 10^6$	$63 \times 10^6$
$k$ ( $\text{W/(m-K)}$ )	401	429

Determine the allowed range of the diameter of the copper core.

*Solution.*

$$\begin{aligned}
k_c &= f_{\text{Cu}} k_{\text{Cu}} + f_{\text{Ag}} k_{\text{Ag}} & \sigma_c &= f_{\text{Cu}} \sigma_{\text{Cu}} + f_{\text{Ag}} \sigma_{\text{Ag}} \\
410 &< f_{\text{Cu}}(401) + (1 - f_{\text{Cu}})(429) & 60 \times 10^6 &< f_{\text{Cu}}(59 \times 10^6) + (1 - f_{\text{Cu}})(63 \times 10^6) \\
f_{\text{Cu}} &< 0.679 & f_{\text{Cu}} &< 0.75
\end{aligned}$$

$$\begin{aligned}
V &= \pi \left( \frac{d}{2} \right)^2 l = \frac{\pi}{4} (1)(100) = 78.5 \text{ cm}^3 \implies \rho_c = \frac{m}{V} = \frac{750}{78.5} = 9.55 \text{ g/cm}^3 \\
9.55 &> f_{\text{Cu}}(8.96 + (1 - f_{\text{Cu}})(10.49)) \implies f_{\text{Cu}} > 0.61
\end{aligned}$$

Taking  $k$ ,  $\sigma$ , and  $V$  into account,  $0.61 < f_{\text{Cu}} < 0.679$ . Thus,

$$f_{\text{Cu}} = \frac{V_{\text{Cu}}}{V_{\text{wire}}} = \frac{d_{\text{Cu}}^2}{d_{\text{wire}}^2} \implies \sqrt{0.61} < d_{\text{Cu}} < \sqrt{0.679} \implies 0.78 \text{ cm} < d_{\text{Cu}} < 0.82 \text{ cm}$$

**Example 1.5.** (Fiber composites).

- *Metal-matrix composites.* Strengthened by metal or ceramic fibers. Provides high temp resistance. E.g., superconducting wire for fusion reactors.
- *Ceramic-matrix composites.* Contain ceramic fibers in ceramic matrix.

**Example 1.6.** (Laminar composites).

- *Laminates.* Layers of materials joined by organic adhesive.
- *Clad materials.* Metal-metal composites. Provide combo of good corrosion resistance w/ high strength.
- *Multilayer capacitors.* Laminar geometry used to make huge numbers of multilayer capacitors.

## 2 Electronic Materials

**Definition 2.1.** (Ohm's law, power, and more).

$$V = IR, \quad R = \frac{\rho L}{A} = \frac{L}{\sigma A} \quad (4)$$

$$P = IV = I^2 R \quad (5)$$

$$J = \frac{I}{A} = \frac{\sigma V}{L} = \sigma E = nq\nu, \quad \nu = l/t \quad (6)$$

$$\mu = \frac{\nu}{E} = \frac{\sigma}{nq} \quad (7)$$

where  $J$  is current density (A/cm<sup>2</sup>),  $E$  is electric field (V/cm),  $n$  is number of charge carriers (carriers/cm<sup>3</sup>),  $q$  is charge on each carrier,  $\nu$  is the average drift velocity (cm/s) and  $\mu$  is mobility (cm<sup>2</sup>/(V·s)).

**Example 2.2.** Design an electrical transmission line 1500 m long with a diameter of 1 cm that will carry a current of 50 A with no more than  $5 \times 10^5$  W power loss.

*Solution.* I.e., what is the minimum conductivity that satisfies these properties?

$$P = I^2 R, \quad R = \frac{1}{\sigma} \left( \frac{L}{\pi(d/2)^2} \right)$$

$$\sigma = \frac{I^2}{P} \left( \frac{4L}{\pi d^2} \right) = \frac{50^2}{5 \times 10^5} \left( \frac{4 \times 1500}{\pi 0.01^2} \right) = 9.54 \times 10^4 \, \Omega^{-1} \cdot \text{m}^{-1}$$

**Definition 2.3.** (Conductivity of metals and alloys). Conductivity defined by the structure of the material iff material is pure and defect-free.

$$\sigma = nq\mu \quad \text{and} \quad \lambda_e = \tau\nu \quad (8)$$

where  $\lambda_e$  is the mean free path and  $\tau$  is the avg time btw collisions. Furthermore,

$$\frac{1}{\sigma} = \rho = \rho_0[1 + \alpha_R(T - 25 \, ^\circ\text{C})] \quad (9)$$

where  $\rho_0$  is resistivity at room temp ( $\Omega \cdot \text{cm}$ ),  $\alpha_R$  is temp resistivity coeff ( $^\circ\text{C}^{-1}$ ), and  $T$  is temp. If there are defects:

$$\rho_d = bx(1 - x) \implies \rho = \rho_t + \rho_d \quad (10)$$

where  $\rho_d$  is resistivity due to defects ( $\Omega \cdot \text{cm}$ ),  $\rho_t$  is resistivity for perfect crystal structure ( $\Omega \cdot \text{cm}$ ),  $b$  is defect resistivity coeff ( $\Omega \cdot \text{cm}$ ), and  $x$  is fraction of impurity.

**Definition 2.4.** (Semiconductors). Energy gap  $E_g$  btw valence and conduction bands is small.

- *Intrinsic semiconductors.* Properties indep of impurities. Pure semiconductors. Temp-dependent. Data gathering and academic applications.
- *Extrinsic semiconductors.* Temp stable and can be ctrl'd by impurities called dopants. Doped semiconductors. Effect of T insignificant. Industrial applications.

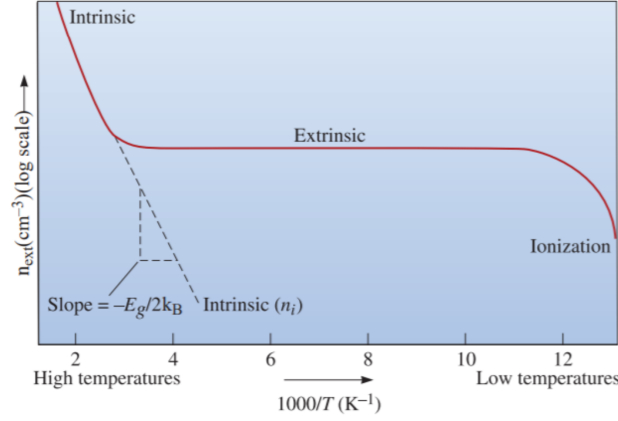
**Definition 2.5.** (Intrinsic semiconductors). For every  $e^-$  promoted to conduction band, there is a hole left in valence band.

$$\sigma = n_i q(\mu_n + \mu_p) \quad (11)$$

where  $n_i$  is conc of  $e^-$  (number of carriers),  $\mu_n$  is mobility of  $e^-$ , and  $\mu_p$  is mobility of holes. In general,  $\mu_n > \mu_p$ . The temperature effect can be described as follows:

$$n_i = n_0 \exp\left(-\frac{E_g}{2k_B T}\right), \quad \text{where } n_0 = 2 \left(\frac{2\pi k_B T}{h^2}\right)^{3/2} (m_n^* m_p^*)^{3/4} \quad (12)$$

$$\sigma = n_0 q(\mu_n + \mu_p) \exp\left(-\frac{E_g}{2k_B T}\right) \quad (13)$$



where  $E_g$  is bandgap,  $k_B$  is Boltzmann's constant ( $1.38 \times 10^{-23} \text{ m}^2 \cdot \text{kg} \cdot \text{s}^{-2} \cdot \text{K}^{-1}$  or  $8.63 \times 10^{-5} \text{ eV/K}$ ),  $h$  is Planck's constant ( $6.63 \times 10^{-34} \text{ m}^2 \cdot \text{kg/s}$ ),  $m_n^*$  is effective mass of  $e^-$ , and  $m_p^*$  is effective mass of holes.

**Example 2.6.** For Ge at 25 °C, find the number of charge carriers/cm<sup>3</sup>, the fraction of the total number of  $e^-$  in the valence band that are excited into the conduction band, and the constant  $n_0$  given

- $\rho_0(\text{Ge}) = 43 \Omega \cdot \text{cm}$
- $\mu_n = 3900 \text{ cm}^2/(\text{V} \cdot \text{s})$
- $a_0 = 5.66 \times 10^{-8} \text{ cm}$
- $\sigma_{\text{RT}}(\text{Ge}) = 0.0233 \Omega^{-1} \cdot \text{cm}^{-1}$
- $\mu_p = 1900 \text{ cm}^2/(\text{V} \cdot \text{s})$
- $\text{DC} \Rightarrow 8 \text{ at/uc.}$
- $E_g = 0.67 \text{ eV}$
- $2k_B T = 0.0514 \text{ eV}$
- Oxidation number = 4.

**TABLE 19-5** ■ Properties of commonly encountered semiconductors at room temperature

Semiconductor	Bandgap (eV)	Mobility of Electrons ( $\mu_n$ ) ( $\frac{\text{cm}^2}{\text{V} \cdot \text{s}}$ )	Mobility of Holes ( $\mu_p$ ) ( $\frac{\text{cm}^2}{\text{V} \cdot \text{s}}$ )	Dielectric Constant (k)	Resistivity ( $\Omega \cdot \text{cm}$ )	Density ( $\frac{\text{g}}{\text{cm}^3}$ )	Melting Temperature (°C)
Silicon (Si)	1.11	1350	480	11.8	$2.5 \times 10^5$	2.33	1415
Amorphous Silicon (a:Si:H)	1.70	1	$10^{-2}$	$\sim 11.8$	$10^{10}$	$\sim 2.30$	—
Germanium (Ge)	0.67	3900	1900	16.0	43	5.32	936
SiC ( $\alpha$ )	2.86	500		10.2	$10^{10}$	3.21	2830
Gallium Arsenide (GaAs)	1.43	8500	400	13.2	$4 \times 10^8$	5.31	1238
Diamond	$\sim 5.50$	1800	1500	5.7	$> 10^{18}$	3.52	$\sim 3550$

*Solution.* Number of carriers:

$$n = \frac{\sigma_{\text{RT}}(\text{Ge})}{q(\mu_n + \mu_p)} = \frac{0.0233 \Omega^{-1} \cdot \text{cm}^{-1}}{(1.6 \times 10^{-19} \text{ C})(3900 \text{ cm}^2/(\text{V} \cdot \text{s}) + 1900 \text{ cm}^2/(\text{V} \cdot \text{s}))} = 2.51 \times 10^{13} \text{ e}^-/\text{cm}^3$$

Fraction of total  $e^-$  in valence band excited into conduction band:

$$n_t = \frac{\text{at/uc} \times \text{oxidation number}}{a_0^3} = \frac{8 \times 4}{(5.66 \times 10^{-8} \text{ cm})^3} = 1.77 \times 10^{23} \text{ e}^-/\text{cm}^3$$

$$\frac{n}{n_t} = \frac{2.51 \times 10^{13} \text{ e}^-/\text{cm}^3}{1.77 \times 10^{23} \text{ e}^-/\text{cm}^3} = 1.42 \times 10^{-10}$$

The constant  $n_0$ :

$$n_0 = \frac{n}{\exp(-E_g/(2k_B T))} = \frac{2.51 \times 10^{13} \text{ e}^-/\text{cm}^3}{\exp(-0.67 \text{ eV}/0.0514 \text{ eV})} = 1.14 \times 10^{19} \text{ e}^-/\text{cm}^3$$

**Definition 2.7.** (Extrinsic semiconductors). Conductivity based on dopants and can be indep of temp.

- *n-type*. Valence of dopant greater than 4 (Fermi lvl closer to conduction band). Donor level  $E_d$  just below Fermi lvl.

$$\sigma = n_i q \mu_n \quad (14)$$

- *p-type*. Valence of dopant less than 4. (Fermi lvl closer to valence band). Acceptor level  $E_a$  just below Fermi lvl.

$$\sigma = n_i q \mu_p \quad (15)$$

**Example 2.8.** Det  $\sigma$  of Si when 0.0001 at% Sb is added as a dopant. Compare it to  $\sigma$  when the same percentage of In is added.

*Solution.*

**TABLE 19-6** ■ The donor and acceptor energy levels (in electron volts) when silicon and germanium semiconductors are doped

Dopant	Silicon		Germanium	
	$E_d$	$E_a$	$E_d$	$E_a$
P	0.045		0.0120	
As	0.049		0.0127	
Sb	0.039		0.0096	
B		0.045		0.0104
Al		0.057		0.0102
Ga		0.065		0.0108
In		0.160		0.0112

Sb:  $\sigma = nq(\mu_n + \mu_p) = nq\mu_n$ , n-type

$$n = \frac{(\text{at/uc})(\text{Sb carriers/at Si})}{a_0^3}$$

$$n = \frac{8(0.0001 \times 10^{-2})}{(5.43 \times 10^{-8} \text{ cm})^3} = 5 \times 10^{16} \text{ Sb carriers/cm}^3$$

$$\sigma = (5 \times 10^{16})(1.6 \times 10^{-19})(1350) = 10.79 \Omega^{-1} \cdot \text{cm}^{-1}$$

In:  $\sigma = nq(\mu_n + \mu_p) = nq\mu_p$ , p-type

$$n = \frac{(\text{at/uc})(\text{In carriers/at Si})}{a_0^3} = \frac{8(0.0001 \times 10^{-2})}{(5.43 \times 10^{-8} \text{ cm})^3} = 5 \times 10^{16} \text{ In carriers/cm}^3$$

$$\sigma = (5 \times 10^{16} \text{ In carriers/cm}^3)(1.6 \times 10^{-19} \text{ C})(480 \text{ cm}^2/(\text{V} \cdot \text{s})) = 3.84 \Omega^{-1} \cdot \text{cm}^{-1}$$

**Definition 2.9.** (Bandgap semiconductors).

- *Direct BG semiconductor (DBG)*. Max E lvl of valence aligns w/ min E lvl of conduction band. DBG always preferred over iBG for making optical sources (e.g., GaAs).
- *Indirect BG semiconductor (IBG)*. Misaligned E lvls (e.g., Si, Ge).

**Formula 2.10.** (Conductivity in ceramics). Unlike previously, entire ions move, not just  $e^-$ .

$$\mu = \frac{zqD}{k_B T} \quad \text{and} \quad \sigma = n_i z q \mu \quad \text{and} \quad D = D_0 \exp\left(-\frac{Q}{RT}\right) \quad (16)$$

where  $z$  is the valence of the ion,  $q$  is the charge on each carrier,  $D$  is the diffusion coeff, and  $n_i$  is the conc of diffusing ions.

**Example 2.11.** Suppose that  $\sigma$  of MgO is determined primarily by the diffusion of  $\text{Mg}^{2+}$  ions. Estimate the mobility of the  $\text{Mg}^{2+}$  ions and calculate the electrical conductivity of MgO at 1500 °C given

- $D_0 = 0.7005 \times 10^{-13} \text{ cm}^2/\text{s}$
- $a_0 = 3.96 \times 10^{-8} \text{ cm}$
- $z = 2$  for  $\text{Mg}^{2+}$
- $Q = 1.25 \times 10^5 \text{ J}$
- $T = 1500^\circ\text{C} = 1773 \text{ K}$
- $\text{MgO}$  has 4 Mg ions/uc

*Solution.*

$$D = D_0 \exp\left(-\frac{Q}{RT}\right) = (0.7005 \times 10^{-13}) \exp\left(-\frac{1.25 \times 10^5}{(8.314)(1773)}\right) = 1.46 \times 10^{-17} \text{ cm}^2/\text{s}$$

$$\mu = \frac{zqD}{k_B T} = \frac{2(1.6 \times 10^{-19})(1.46 \times 10^{-17})}{(1.38 \times 10^{-23})(1773)} = 1.909 \times 10^{-16} \text{ cm}^2/(\text{V}\cdot\text{s})$$

$$n(\text{Mg}^{2+}) = \frac{4}{(3.96 \times 10^{-8})^3} = 6.4 \times 10^{22} \text{ ions/cm}^3$$

$$\sigma = n z q \mu = (6.4 \times 10^{22})(2)(1.6 \times 10^{-19})(1.9 \times 10^{-16}) = 3.9 \times 10^{-12} \Omega^{-1}\cdot\text{cm}^{-1}$$

**Formula 2.12.** (Dielectric and insulating materials: polarization).

$$U = qd \quad \text{and} \quad P = zqd$$

where  $P$  is polarization ( $\text{C/m}^2$ ),  $z$  is the number of charges displaced per unit volume,  $d$  is avg displacement (m).

**Example 2.13.** Calculate the displacement of  $e^-$  if the polarization of Al is  $2 \times 10^{-8} \text{ C/m}^2$  and given  $a_0 = 4.04 \times 10^{-8} \text{ cm}$ , and Al has FCC structure.

*Solution.*

$$z = \frac{\text{at/uc} \times \text{atomic number}}{a_0^3} = \frac{4 \times 13}{(4.04 \times 10^{-8})^3} = 7.89 \times 10^{23} \text{ e}^-/\text{cm}^3 = 7.89 \times 10^{29} \text{ e}^-/\text{m}^3$$

$$d = \frac{P}{zq} = \frac{2 \times 10^{-8}}{(7.89 \times 10^{29})(1.6 \times 10^{-19})} = 1.58 \times 10^{-19} \text{ m}$$

**Formula 2.14.** (Capacitors).  $Q = CV$ ,  $\kappa = \epsilon/\epsilon_0$ ,  $C = \epsilon A/d = \kappa \epsilon_0 A/d$ ,  $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$ .

**Definition 2.15.** (Linear and non-linear dielectrics).

- *Linear.* Polarization only occurs when an E field is applied.  $P = (k - 1)\epsilon_0 E$ ,  $V = Ed$ .  $\chi = k - 1$  is dielectric susceptibility.
- *Non-linear.* Polarization remnant even after E field removed.
  - Electrostricticity: dimensional change in material when there is E.
  - Piezoelectricity: application of stress produces polarization.
  - Ferroelectricity: spontaneous and reversible dielectric polarization.

**Definition 2.16.** (Superconductivity). Below  $T_c$ , zero resistance, Meissner effect occurs.

- *Type I.* Most ideal metals. Completely expels B field.
- *Type II.* Intermetallic compounds. Able to lose superconductivity.

### 3 Magnetic Materials

**Definition 3.1.** (Classification of magnetic response).

Soft	Hard
Easily magnetized	Almost impossible to be magnetized
Can lose magnetic behav	Don't lose magnetic behav
Large values for susceptibility and permeability	Small values for susceptibility and permeability
Electromagnets	Permanent magnets
Fe-Si, Fe-Ni, ferrites	Fe-Ni-Al, Co alloys

**Definition 3.2.** (Magnetic field, permeability, and magnetization). Magnetic field is defined as

$$H = \frac{nI}{L} \quad (17)$$

where  $n$  is number of turns,  $L$  is length of coil (m),  $I$  is current (A), and  $H$  is magnetic field (A/m or oersted;  $4\pi \times 10^{-3}$  oersted = 1 A/m). Relative permeability  $\mu_r$  is defined as a ratio of permeability

$$\mu_r = \frac{\mu}{\mu_0} \quad (18)$$

where  $\mu > \mu_0$  if magnetic moments in same dir of applied field and  $\mu < \mu_0$  if magnetic moments oppose the field ( $\mu_0 = 4\pi \times 10^{-7}$  H/m). Inductance  $B$  (H or Teslas) is defined as

$$B = \mu_0 H, \quad \text{in vacuum} \quad (19)$$

$$B = \mu(H + M), \quad \text{in material} \quad (20)$$

$$M = \frac{(\text{at/uc})(\text{magnetron/at})\mu_B}{a_0^3} \quad (21)$$

$$X_m = M/H = \mu_r - 1 \quad (22)$$

where  $M$  is magnetization (A/m),  $\mu_B$  is Bohr magneton ( $9.27 \times 10^{-24}$  A·m<sup>2</sup>/magnetron), and  $X_m$  is magnetic susceptibility. We can use oxidation number to estimate magnetron/at.

**Example 3.3.** Estimate the magnetization produced in a bar made of Ni given  $A_0(\text{Ni}) = 3.52 \times 10^{-10}$  m, oxidation number is 2, and FCC.

*Solution.*

$$M = \frac{(\text{at/uc})(\text{magnetron/at})\mu_B}{a_0^3} = \frac{4 \times 2 \times (9.27 \times 10^{-24})}{(3.52 \times 10^{-10})^3} = 1.7 \times 10^6 \text{ A/m}$$

**Example 3.4.** 0.0015 at% of Ni is inserted into Cu. This material has max permeability of  $4.5 \times 10^{-3}$  H/m when inductance of  $3.5 \text{ A} \cdot \text{H/m}^2$  is obtained. The alloy is placed in 20-turn coil 20 cm in length. What current must flow through the conductor to obtain this field given  $a_0(\text{Ni}) = 3.52 \times 10^{-10}$  m,  $a_0(\text{Cu}) = 3.61 \times 10^{-10}$  m?

*Solution*

$$a_0 = f_{\text{matrix}}a_{0,\text{matrix}} + f_{\text{solute}}a_{0,\text{solute}} \approx 3.61 \times 10^{-10} \text{ m}$$

$$M = \frac{(\text{at/uc})(f \text{ of solute})(\text{magnetron/at of solute})\mu_B}{a_0^3}$$

$$= \frac{4(0.0015 \times 10^{-2})(2)(9.27 \times 10^{-24})}{(3.61 \times 10^{-10})^3}$$

$$= 23.64 \text{ A/m}$$

$$B = \mu(H + M) = \mu\left(\frac{nI}{L} + M\right) \implies I = \left(\frac{B}{\mu} - M\right) \frac{L}{n} = \left(\frac{3.5}{4.5 \times 10^{-3}} - 23.64\right) \frac{0.2}{20} = 7.54 \text{ A}$$

**Definition 3.5.** (Classification of magnetic response).

Diamagnetism	Paramagnetism	Ferromagnetism	Ferrimagnetism
Induced opp mag dipole	Induced random mag dipole	Induced mag dipole	Induced mag dipole both parallel and opposite
Opposing $H$	No interaction among dipoles	Amplifies $H$	Amplifies $H$
Au, Ag, Cu, Hg	Ca, Al, Cr	Fe, Co	Zn, Ni, ceramics

## 4 Photonic Materials

**Definition 4.1.** (Intensity). Intensity is the sum of reflection, absorption, and transmission.

$$I_0 = I_r + I_a + I_t \quad (23)$$

**Definition 4.2.** (Refraction).

$$n = \frac{c_0}{c} = \frac{\lambda_{\text{vacuum}}}{\lambda} = \frac{\sin \theta_i}{\sin \theta_t} \quad (24)$$

$$c = \frac{1}{\sqrt{\mu\epsilon}}, \quad n = \frac{\sqrt{\mu\epsilon}}{\sqrt{\mu_0\epsilon_0}} \approx \sqrt{\kappa} \quad (25)$$

$$\frac{c_1}{c_2} = \frac{n_2}{n_1} = \frac{\sin \theta_1}{\sin \theta_2} \quad (26)$$

where  $\kappa$  is the dielectric constant.

**Definition 4.3.** (Reflection). Occurs at interface btw 2 materials.

$$R = \left( \frac{n - 1}{n + 1} \right)^2, \quad \text{in air} \quad (27)$$

$$R = \left( \frac{n - n_i}{n + n_i} \right)^2, \quad \text{in other materials} \quad (28)$$

$$I_r = RI_0 \quad (29)$$

**Definition 4.4.** (Absorption).

$$I = I_0 e^{-\alpha x} \quad (30)$$

where  $\alpha$  is the linear absorption coefficient and  $x$  is the photon's path (often the thickness of the material).

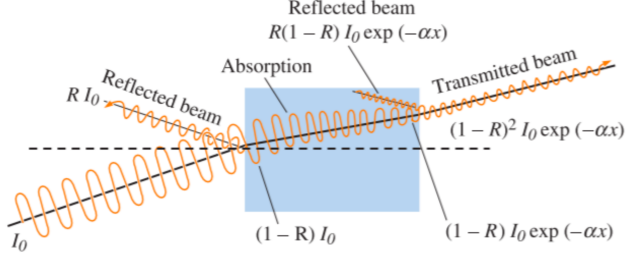
**Definition 4.5.** (Transmission). Depends on properties of material and photon's wavelength.

- *Microstructure.* Higher pf and larger atoms transmit less.
- *Porosity.* More porous means more transmission.
- *Bandgap.* Less likely in metals b/c of overlapping bands. Insulators and semiconductors have increased transmission.

**Transmission formulas:**

1. After 1st reflection:  $I_1 = (1 - R)I_0$
2. After absorption:  $I_2 = I_1 e^{-\alpha x} = (1 - R)I_0 e^{-\alpha x}$
3. After 2nd reflection:  $I_{rb} = RI_2 = R(1 - R)I_0 e^{-\alpha x}$
4. Difference btw absorption and 2nd reflection:  $I_t = (1 - R)^2 I_0 e^{-\alpha x}$





**Definition 4.6.** (Reflection, Absorption, Transmission).

$$R = \frac{I_r}{I_0} = \left( \frac{n-1}{n+1} \right)^2 \quad (31)$$

$$A = 1 - (1-R)e^{-\alpha x} \quad (32)$$

$$T = (1-R)^2 e^{-\alpha x} \quad (33)$$

$$1 = T + R + A, \quad \text{if } 0.01 < R < 0.05 \quad (34)$$