EN PH 131 Formula Sheet

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Conversion Factors and Constants

Quantity	FPS Unit	SI Unit
Force	1 lb	4.4482 N
Mass	1 slug	14.5938 kg
Length	1 ft	0.3048 m
	1 in	0.0254 m

Distance: 1 ft = 12 in — 1 mi = 5280 ft

Weight: 1 kip = 1000 lb — 1 ton = 2000 lb

$\text{slug} = \text{lb}\cdot\text{s}^2\cdot\text{ft}^{-1}$ — $g = 9.81\text{ m/s}^2 = 32.2\text{ ft/s}^2$

$G = 6.673\times10^{-12}\text{ Nm}^2/\text{kg}^2$ — 1 hp = 550 lb·ft/s = 746 W

Graph Interpretation

Graph	Slope interpretation	Integral interpretation
s-t	velocity $\left(\frac{ds}{dt}\right)$	
v-t	acceleration $\left(\frac{dv}{dt}\right)$	$\Delta s = \int v\ dt$
a-t	jerk $\left(\frac{da}{dt}\right)$	$\Delta v = \int a\ dt$
v-s	acceleration $\left(a = v\frac{dv}{ds}\right)$	
a-s		$\int_{v_0}^{v} v\ dv = \int_{s_0}^{s} a\ ds$

Rectilinear Motion

$v = \frac{ds}{dt}$ $\qquad a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$ $\qquad a\ ds = v\ dv$

$\int_{v_0}^{v} dv = a_c\int_0^t dt$ $\qquad v = v_0 + a_c t$

$\int_{s_0}^{s} ds = \int_0^t (v_0 + a_c t)\ dt$ $\qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$\int_{v_0}^{v} v\ dv = a_c\int_{s_0}^{s} ds$ $\qquad v^2 = v_0^2 + 2a_c(s - s_0)$

Relative Motion

$\mathbf{r}_{B/A} = \mathbf{r}_B - \mathbf{r}_A$ $\qquad \mathbf{v}_{B/A} = \mathbf{v}_B - \mathbf{v}_A$ $\qquad \mathbf{a}_{B/A} = \mathbf{a}_B - \mathbf{a}_A$

$\mathbf{r}_{B/A} = -\mathbf{r}_{A/B}$ $\qquad \mathbf{v}_{B/A} = -\mathbf{v}_{A/B}$ $\qquad \mathbf{a}_{B/A} = -\mathbf{a}_{A/B}$

B/A: B with respect to A

Miscellaneous

$f_{\text{res}} = -ks$ $\qquad k_{\text{eff}} = \sum_{i=1}^{n} k_i$ (parallel) $\qquad k_{\text{eff}} = \left(\sum_{i=1}^{n}\frac{1}{k_i}\right)^{-1}$ (series)

$f_k = \mu_k N$ $\qquad f_s \le \mu_s N$ $\qquad \cos\theta = \frac{\mathbf{a}\cdot\mathbf{b}}{ab}$

Curvilinear Motion

$s = \int_{t_0}^{t} |\mathbf{r}'(t)|\ dt$ $\qquad \mathbf{v} = \langle\dot{x},\dot{y},\dot{z}\rangle$ $\qquad \mathbf{a} = \langle\ddot{x},\ddot{y},\ddot{z}\rangle$

$\mathbf{a} = a_t\mathbf{u}_t + a_n\mathbf{u}_n$ $\qquad a_t = \dot{v}$ $\qquad a_n = v^2/\rho$ $\qquad a = \sqrt{a_t^2 + a_n^2}$

$\sum F_t = ma_t$ $\qquad \sum F_n = ma_n = \frac{mv^2}{\rho}$ $\qquad \sum F_b = 0$

$\rho(x) = \frac{(1 + [f'(x)]^2)^{3/2}}{|f''(x)|}$ $\qquad \rho(t) = \frac{|r'(t)|^3}{|r'(t)\times r''(t)|} = \frac{(\dot{x}^2 + \dot{y}^2)^{3/2}}{|\dot{x}\ddot{y} - \ddot{x}\dot{y}|}$

- $a = a_t$ if straight line ($\rho = \infty$)
- $a = a_n$ if v constant on curve ($a_t = \dot{v} = 0$)

Work and Energy

$U_{1-2} = \int_{\mathbf{r}_1}^{\mathbf{r}_2}\mathbf{F}\cdot d\mathbf{r} = \int_{s_1}^{s_2} F\cos\theta\ ds$

Principle of work and energy: $\sum U_{1-2} = T_2 - T_1$

Conservation of energy: $\sum T_1 + \sum V_1 = \sum T_2 + \sum V_2$

Work $= \Delta T = -\Delta V$

Compressed: $f_s = k(s_0 - s)$ — Stretched: $f_s = k(s - s_0)$

Work type	Integral
Straight line	$U_{1-2} = F\cos\theta\int_{s_1}^{s_2} ds = F\cos\theta(s_2 - s_1)$
Weight	$U_{1-2} = -W\int_{y_1}^{y_2} dy = -W(y_2 - y_1)$
Spring force	$U_{1-2} = -k\int_{s_1}^{s_2} s\ ds = -\frac{1}{2}k(s_2^2 - s_1^2)$

Power and Efficiency

$P = \frac{dU}{dt} = \frac{\mathbf{F}\cdot d\mathbf{r}}{dt} = \mathbf{F}\cdot\mathbf{v}$ $\qquad \epsilon = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{E_{\text{out}}}{E_{\text{in}}}$

$U_{\text{out}} = \epsilon\int P_{\text{in}}\ dt$ $\qquad \mathbf{F} = -\nabla V$

Linear Impulse and Momentum

Single particle: $\int_{t_1}^{t_2}\mathbf{F}\ dt = m\int_{v_1}^{v_2} d\mathbf{v} = m\mathbf{v}_2 - m\mathbf{v}_1$

System of particles: $\sum_{i=1}^{n}\int_{t_1}^{t_2}\mathbf{F}\ dt = \sum_{i=1}^{n} m_i(\mathbf{v}_2 - \mathbf{v}_1) = m\mathbf{v}_{g2} - m\mathbf{v}_{g1}$

Linear impulse (action): $\mathbf{I} = \int_{t_1}^{t_2}\mathbf{F}\ dt$ — N·s or lb·s

Linear momentum (response): $\mathbf{L} = m\mathbf{v}$ — kg·m/s or slug·ft/s

Conservation of linear momentum: $\sum m_i\mathbf{v}_{i1} = \sum m_i\mathbf{v}_{i2}$

$e = \frac{(v_{b/a})_2}{(v_{a/b})_1} = \frac{v_{b2} - v_{a2}}{v_{a1} - v_{b1}} = \frac{\text{restitution impulse}}{\text{deformation impulse}}$

$e = -\frac{v_{a2}}{v_{a1}}$ if obj b doesn't move after impact

Oblique impact: y-cmpt conserved; need e for x-cmpt

Angular Motion

$\omega = \frac{d\theta}{dt}$ $\alpha = \frac{d\omega}{dt} = \frac{d\theta}{dt}$ $\alpha d\theta = \omega d\omega$

$\omega = \omega_0 + \alpha t$ $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

$s = r\theta$ $v = r\omega$ $a_t = r\alpha$ $a_n = r\omega^2$

$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$ $\mathbf{a} = \boldsymbol{\alpha} \times \mathbf{r} - \omega^2 \mathbf{r}$

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Rigid Body Motion

$\mathbf{M} = \mathbf{r} \times \mathbf{F}$ $I = \int_m r^2 \, dm = \int_V r^2 \rho \, dV$ $I = I_{\text{cm}} + md^2$

Object	I
Thin uniform rod, axis about center	$ML^2/12$
Thin uniform rod, axis about one end	$ML^2/3$
Uniform solid cylinder/disk, axis thru centre	$MR^2/2$
Uniform hollow cyilnder/hoop axis thru centre	MR^2
Uniform solid sphere, axis thru centre	$2MR^2/5$
Uniform hollow sphere, axis thru centre	$2MR^2/3$

Eqs of motion: $\begin{cases} \sum \mathbf{F} = m\mathbf{a}_{\text{cm}} \\ \sum \mathbf{M}_0 = I_0 \boldsymbol{\alpha} \end{cases}$ $\begin{cases} a_{\text{cm}} > r\alpha & \text{slipping} \\ a_{\text{cm}} = R\alpha & \text{no slipping} \end{cases}$

$T = \frac{1}{2}mv_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}\omega^2$ $W_{\text{rot}} = \int_{\theta_1}^{\theta_2} \tau \, d\theta$