ECE 240 Formula Sheet

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Properties of Signals

$f_e(t) \iff f(t) = f(-t)$ $\qquad$ $f_0(t) \iff f(t) = -f(-t)$

$\int_{-a}^{a} f_e(t)\ dt = 2\int_0^a f_e(t)\ dt$ $\qquad$ $\int_{-a}^{a} f_o(t)\ dt = 0$

$f_e(t) + f_e(t) = f_e(t)$ $\qquad$ $f_o(t) + f_o(t) = f_o(t)$

$f_e(t)f_e(t) = f_e(t)$ $\qquad$ $f_o(t)f_o(t) = f_e(t)$ $\qquad$ $f_e(t)f_o(t) = f_o(t)$

$\forall f,\ \exists f(t)$ s.t. $f(t) = f_e(t) + f_o(t)$

$$f_e(t) = \frac{1}{2}[f(t) + f(-t)] \qquad f_o(t) = \frac{1}{2}[f(t) - f(-t)]$$

Classifying Signals

Energy signal: $E = \int_{-\infty}^{\infty} |f(t)|^2\ dt < \infty$ $\qquad$ $|f(t)|^2 = f(t)\overline{f(t)}$

Power signal (non-periodic): $P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |f(t)|^2\ dt$

Power signal (periodic): $P = \frac{1}{T}\int_{-T/2}^{T/2} |f(t)|^2\ dt$ $\qquad$ $0 < P < \infty$

$$f(t) = \sum_{i=1}^{k} A_i \cos(\omega_i t + \phi_i) \implies P = \frac{1}{2}\sum_{i=1}^{k} A_i^2$$

If a is period of f and b is period of g, period of $f+g$ is $\mathrm{LCM}(a,b)$

$f(t+T) = f_1(t + k_1T_1) + f_2(t + k_2T_2)$ periodic iff $\frac{k_1}{k_2} = \frac{T_2}{T_1}$

$T = k_1T_1 = k_2T_2$

Signal Operations

Time reversal: $y(t) = f(-t)$

Time shift: $y(t) = f(t-b)$ $\qquad$ $b > 0$: right, $b < 0$: left

Time scale: $y(t) = f(at)$

$|a| < 1$: stretch, $|a| > 1$: compress, $|a| < 0$: reflect over y-axis

Combined ops: $y(t) = f(at - b)$

Time shift by b then scale by a OR scale by a then shift by b/a

Elementary Signals

Heaviside: $H(t) = \frac{1}{2} + \frac{1}{\pi}\arctan(\lambda t)$

Unit step: $u(t) = \lim_{\lambda\to\infty} H(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$

$$\int_{-\infty}^{t} u(t-T)\ dt = \left(\int_T^t dt\right) u(t-T)$$

Rectangular pulse: $p_d(t) = u(t + d/2) - u(t - d/2) = \begin{cases} 1, & |t| < d/2 \\ 0, & |t| > d/2 \end{cases}$

Unit impulse: $\delta(t) = \lim_{\sigma\to 0} \frac{dH(t)}{dt} = \frac{du(t)}{dt}, \quad \sigma = 1/\lambda$

$\int_{-\infty}^{\infty} \delta(t)\ dt = 1, \quad \delta(t) = 0,\ \forall t \neq 0$ $\qquad$ $\phi(t)\delta(t-T) = \phi(T)\delta(t-T)$

Sampling: $\int_{-\infty}^{\infty} \phi(t)\delta(t-T)\ dt = \int_{-\infty}^{\infty} \phi(t)\delta(T-t)\ dt = \phi(T)$

$\int_{-\infty}^{t} \delta(t-T)\ dt = u(t-T)$ $\qquad$ $\delta(at) = \frac{1}{|a|}\delta(t) \implies \delta(t) = \delta(-t)$

$\mathrm{sinc}(t) = \frac{\sin t}{t}$, $\quad$ zeroes at $n\pi,\ n \in \mathbb{Z}$

Complex exponential: $f(t) = Ae^{st}$, $\quad s = \sigma + j\omega, A = |A|e^{j\theta}$

$f(t) = |A|e^{\sigma t}(\cos(\omega t + \theta) + j\sin(\omega t + \theta))$

$\sigma < 0$: decaying sinusoid, $\sigma = 0$: sinusoid, $\sigma > 0$: growing sinusoid

Continuous-Time Signal Properties

$f(t) \longrightarrow \boxed{\mathcal{H}} \longrightarrow y(t)$ $\qquad$ $\mathcal{H}$ is the system

$\mathcal{H}$ is memoryless iff $y(t_0) = H[f(t_0)]$, else has memory.

$\mathcal{H}$ is causal iff $y(t_0)$ is a fn of $f(t)$ for $t < t_0$ (not future values).

$\mathcal{H}$ is linear iff $a_1f_1 + a_2f_2 \xrightarrow{\mathcal{H}} a_1y_1 + a_2y_2$. That is, given $f_1(t) \xrightarrow{\mathcal{H}} y_1(t)$ and $f_2(t) \xrightarrow{\mathcal{H}} y_2(t)$, superposition and homogeneity are satisfied.

1. Is $\frac{dy}{dt} + 2y = f^2(t)$ linear?
2. $\frac{dy_1}{dt} + 2y_1 = f_1^2 \implies a_1\frac{dy_1}{dt} + 2a_1y_1 = a_1f_1^2$
3. $\frac{dy_2}{dt} + 2y_2 = f_2^2 \implies a_2\frac{dy_2}{dt} + 2a_2y_2 = a_2f_2^2$
4. $\frac{d}{dt}(a_1y_1 + a_2y_2) + 2(a_1y_1 + a_2y_2) = a_1f_1^2 + a_2f_2^2 \neq (a_1f_1 + a_2f_2)^2$ $\qquad$ Thus, $\mathcal{H}$ is nonlinear.

$\mathcal{H}$ is time-invariant iff $f(t) \xrightarrow{\mathcal{H}} y(t) \implies f(t-T) \xrightarrow{\mathcal{H}} y(t-T)$

1. $\mathcal{H}: f \mapsto y,\ y = tf(t)$.
2. Let $y_1(t) = tf_1(t)$, $y_2(t) = tf_2(t)$, and $f_2(t) = f_1(t-T)$.
3. Then $y_2(t) = tf_1(t-T)$ but $y_1(t-T) = (t-T)f_1(t-T)$.
4. Thus, $\mathcal{H}$ is time-varying.

Another example:

1. $\mathcal{H}: f \mapsto y,\ y(t) = f(at)$.
2. $y_1(t-T) = f_1(a(t-T)) = f_1(at - aT)$.
3. $f_2(t) = f_1(t-T) \implies y_2(t) = y_1(at - T)$.
4. Thus, $\mathcal{H}$ is time-varying.

$\mathcal{H}$ is BIBO stable iff $|f(t)| \leq A < \infty \implies |y(t)| \leq B < \infty,\ A, B \in \mathbb{R}$.

Convolutions

Convolution: $f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(\tau)f_2(t-\tau)\ d\tau$

If $h(t) = \mathcal{H}(\delta(t))$, then $y(t) = \mathcal{H}(f(t)) = f(t) * h(t)$. That is,

$\delta(t) \longrightarrow \boxed{\mathcal{H}} \longrightarrow h(t)$ $\qquad$ $f(t) \longrightarrow \boxed{\mathcal{H}} \longrightarrow f(t) * h(t)$

Commutation: $f_1(t) * f_2(t) = f_2(t) * f_1(t)$

Distribution: $f_1 * (f_2 + f_3) = f_1 * f_2 + f_1 * f_3$

Association: $f_1 * (f_2 * f_3) = (f_1 * f_2) * f_3$

Time shifting: $f_1(t - T_2) * f_2(t - T_2) = y(t - T_1 - T_2)$

$f_1(t-T) * f_2(t) = y(t-T)$ $\qquad$ $f_1(t) * f_2(t-T) = y(t-T)$

Convolution w/ unit impulse: $f(t) * \delta(t) = f(t)$

Causal signals: $f_1(t) * f_2(t) = \int_0^t f_1(\tau)f_2(t-\tau)\ d\tau$

LTI System Properties

$\mathcal{H}$ is memoryless iff $h(t) = k\delta(t)$.

$\mathcal{H}$ is causal iff $h(t) = 0 \ \forall t < 0$.

$\mathcal{H}$ is is BIBO stable iff $h(t)$ is absolutely integrable, i.e.,

$$\int_{-\infty}^{\infty} |h(t)| \ dt \leq M < \infty$$

Inner Products & Orthogonality

Inner product: $\langle f_1, f_2 \rangle = \int_{t_1}^{t_2} f_1(t)\overline{f_2(t)} \ dt$

Orthogonal over $[t_1, t_2] \iff \langle f_1, f_2 \rangle = 0$

Orthogonal set if $f_1, \ldots, f_n$ over $[t_1, t_2]$ is $\langle f_i, f_j \rangle = \begin{cases} 0, & i \neq j \\ E_{ij}, & i = j \end{cases}$

Let $\mathcal{S} = \{1, \cos\omega_0 t, \ldots, \cos n\omega_t, \ldots, \sin\omega_0 t, \ldots, \sin n\omega_t, \ldots\}$. Then $\mathcal{S}$ is an orthognal set over the interval $[-T/2, T/2]$ iff

1. $\langle \cos n\omega_0 t, \sin k\omega_0 t \rangle = \begin{cases} 0, & n \neq k \\ T/2, & n = k \neq 0 \end{cases}$
2. $\langle \sin n\omega_0 t, \sin k\omega_0 t \rangle = \begin{cases} 0, & n \neq k \\ T/2, & n = k \neq 0 \end{cases}$
3. $\langle 1, \sin n\omega_0 t \rangle = 0, \quad \forall n$
4. $\langle 1, \cos n\omega_0 t \rangle = 0, \quad \forall n$

Trigonometric Fourier Series

$$f(t) = f(t+T) \implies f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} f(t) \ dt \qquad \omega_0 = \frac{2\pi}{T}$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos n\omega_0 t \ dt \qquad b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin n\omega_0 t \ dt$$

$$f(t) \text{ odd } \implies a_0 = a_n = 0 \qquad b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin n\omega_0 t \ dt$$

$$f(t) \text{ even } \implies b_n = 0 \qquad a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos n\omega_0 t \ dt$$

Let $f(t) = f(t+T)$ and $S_k(t) = a_0 + \sum_{n=1}^{k}(a_0 \cos n\omega_0 t + b_n \sin n\omega_0 t)$. Then if $\epsilon_k(t) = f(t) - S_k(t)$,

$$P_{\epsilon_k}(t) = \frac{1}{T}\int_{-T/2}^{T/2} \epsilon_k^2(t) \ dt = \frac{1}{T}\int_{-T/2}^{T/2} f^2(t) \ dt - a_0^2 - \frac{1}{2}\sum_{n=1}^{k}(a_n^2 + b_n^2)$$

$$\frac{1}{T}\int_{-T/2}^{T/2} f^2(t) \ dt \geq a_0^2 + \frac{1}{2}\sum_{n=1}^{k}(a_n^2 + b_n^2) \qquad \lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n = 0$$

If $\int_T |f(t)| \ dt < \infty$ and f has finite number of maxima, minima, discontinuities in each period, then the series converges to $f(t)$ except at discontinuities, where it converges to $\frac{1}{2}[f(t_d^-) + f(t_d^+)]$.

Compact Form

$$f(t) = c_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \theta_n)$$

$$c_0 = a_0 \qquad c_n = \sqrt{a_n^2 + b_n^2} \qquad \theta_n = \arctan(-b_n/a_n), \ \theta_n \in [-\pi, \pi]$$

Exponential Form

$$f(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$$

$$D_n = \frac{a_n - jb_n}{2} \qquad D_{-n} = \frac{a_n + jb_n}{2} \qquad D_0 = a_0$$

$$D_n = \frac{1}{T}\int_T f(t)e^{-jn\omega_0 t} \ dt \qquad D_n = \frac{c_n}{2}e^{j\theta_n}, \quad D_{-n} = \frac{c_n}{2}e^{-j\theta_n}$$

$$|D_n| = \frac{c_n}{2} \implies \text{even fn} \qquad \angle D_n = \theta_n, \quad D_{-n} = -\theta_n \implies \text{odd fn}$$

$$f(t) \text{ even: } |D_n| = \frac{|a_n|}{2}, \quad \angle D_n = \begin{cases} 0, & a_n > 0 \\ \pm\pi, & a_n < 0 \end{cases}$$

$$f(t) \text{ odd: } |D_n| = \frac{|b_n|}{2}, \quad \angle D_n = \begin{cases} \pi/2, & b_n > 0 \\ -\pi/2, & b_n < 0 \end{cases}$$

Parseval's Theorem (Discrete)

$$P_f = a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2) = c_0^2 + \frac{1}{2}\sum_{n=1}^{\infty} c_n^2 = D_0^2 + 2\sum_{n=1}^{\infty}|D_n|^2 = \sum_{n=-\infty}^{\infty}|D_n|^2$$

P_{f_k} is power of FS w/ k terms

Error to $n\%$: $P_{f_k} = (1 - n\%)P_f$ **Caution:** Count the const term!

Identities

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y \qquad e^{j\theta} = \cos\theta + j\sin\theta$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y \qquad \cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\sin \mapsto \cos: \ \sin\theta = \cos(\theta - 90°) \qquad \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$\cos \mapsto \sin: \ \cos\theta = \sin(\theta + 90°)$$

Double Angle Formulas

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

Half Angle Formulas

$$\sin x \sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$$

$$\cos x \cos y = \frac{1}{2}[\cos(x-y) + \cos(x+y)]$$

$$\sin x \cos y = \frac{1}{2}[\sin(x+y) + \sin(x-y)]$$

$$\cos x \sin y = \frac{1}{2}[\sin(x+y) - \sin(x-y)]$$

Fourier Transform

$$F(\omega) = \mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t)e^{-j\omega t} \ dt$$

$$f(t) = \mathcal{F}^{-1}\{\mathcal{F}(\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t} \ d\omega$$

$\mathcal{F}(\omega) = \text{Re}\{F(\omega)\} + j\text{Im}\{F(\omega)\}$ $\qquad$ $|\mathcal{F}(\omega)|$ even, $\angle\mathcal{F}(\omega)$ odd

Frequency Response of LTI Systems

$\delta(t) \longrightarrow \boxed{\mathcal{H}} \longrightarrow h(t)$ $\qquad$ $f(t) \longrightarrow \boxed{\mathcal{H}} \longrightarrow f(t) * h(t)$

Freq response: $H(\omega) = \dfrac{Y(\omega)}{F(\omega)}$

$|Y(\omega)| = |H(\omega)||F(\omega)|$ $\qquad$ $\angle Y(\omega) = \angle H(\omega) + \angle F(\omega)$

$\mathcal{H}$ is distortion-free if $H(\omega) = Ke^{-j\omega t_0}$

Parseval's Theorem (Continuous)

$$E_f = \int_{-\infty}^{\infty} |f(t)|^2 \, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(\omega)|^2 \, d\omega$$

E spectral density: $|F(\omega)|^2$

Time compression $\iff$ spectral expansion

Time expansion $\iff$ spectral compression

LPF: $|H(\omega)| = \begin{cases} e^{-j\omega t_0}, & |\omega| < \omega_c \\ 0, & |\omega| > \omega_c \end{cases}$ $\qquad$ $|H(\omega)| = \begin{cases} 1, & |\omega| < \omega_c \\ 0, & |\omega| > \omega_c \end{cases}$

Impulse Response

$Y(s) = H(s)X(s)$

$x(t) = \delta(t) \implies \mathcal{L}[\delta(t)] = 1 \implies y(t) = \mathcal{L}^{-1}[H(s)] = h(t)$

Given $h(t)$, $y(t) = \mathcal{L}^{-1}[\ \mathcal{L}[h(t)]\mathcal{L}[x(t)]\ ]$

Amplitude Modulation

AM Transmission

$$\mathcal{F}\{f(t)\cos\omega_0 t\} = \frac{1}{2}\left[F(\omega+\omega_0) + F(\omega-\omega_0)\right]$$

$f(t) \longrightarrow \boxed{\text{LPF}} \longrightarrow \boxed{\text{gain: } \cos\omega_c t} \longrightarrow \boxed{\text{antenna}} \longrightarrow$ signal

AM Reception

$$\mathcal{F}\{f(t)\cos^2\omega_0 t\} = \frac{1}{2}F(\omega) + \frac{1}{4}\left[F(\omega+2\omega_0) + F(\omega-2\omega_0)\right]$$

BPF selects the carrier frequency ω_c and LPF removes the $F/4$ terms.

signal $\longrightarrow \boxed{\text{BPF}} \longrightarrow \boxed{\text{gain: } \cos\omega_c t} \longrightarrow \boxed{\text{LPF}} \longrightarrow f(t)$

Unilateral Laplace Transform

$$F(s) = \int_{0^-}^{\infty} f(t)e^{-st} \, dt \qquad f(t) \text{ causal}$$

$$f(t) = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} F(s)e^{st} \, dt$$

LT exists if $|f(t)| \le Me^{\sigma_0 t}$, $\sigma_0 \in \mathbb{R}$

IVT: $f(0^+) = \lim_{s\to\infty} sF(s)$ $\qquad$ FVT: $\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$

Partial Fraction Expansion

Real & distinct roots: $\dfrac{P(s)}{Q(s)} = \dfrac{K_1}{s+p_1} + \cdots + \dfrac{K_j}{s+p_j} + \cdots + \dfrac{K_n}{s+p_n}$

$$K_j = \frac{P(s)}{Q(s)}(s+p_j)|_{s=-p_j}$$

$$\mathcal{L}^{-1}\left[\frac{P(s)}{Q(s)}\right] = u(t)\left(K_1e^{-p_1 t} + \cdots + K_je^{-p_j t} + \cdots + K_ne^{-p_n t}\right)$$

Distinct complex roots: $\dfrac{P(s)}{Q(s)} = \dfrac{K}{s+(\alpha-j\beta)} + \dfrac{K^*}{s+(\alpha+j\beta)} + \cdots$

$$K = \frac{P(s)}{Q(s)}(s+\alpha-j\beta)|_{s=-(\alpha-j\beta)} = |K|\angle\theta_K$$

$$\mathcal{L}^{-1}\left[\frac{P(s)}{Q(s)}\right] = u(t)\left[2|K|e^{-\alpha t}\cos(\beta t+\theta_K) + \cdots\right]$$

Repeated real roots: $\dfrac{P(s)}{Q(s)} = \dfrac{K_1}{(s+p)^n} + \dfrac{K_2}{(s+p)^{n-1}} + \cdots + \dfrac{K_n}{s+p}$

$$K_j = \frac{1}{(j-1)!}\left[\frac{d^{(j-1)}}{ds^{(j-1)}}(s+p)^n F(s)\right]_{s=-p}$$

Zero-Input & Zero-State Response

$$Q(s)Y(s) - N(s) = P(s)F(s) \implies Y(s) = \underbrace{\frac{N(s)}{Q(s)}}_{\text{ICs}} + \underbrace{\frac{P(s)}{Q(s)}F(s)}_{\text{forcing fn}}$$

ZIR: $y_0(t) = \mathcal{L}^{-1}\left\{\dfrac{N(s)}{Q(s)}\right\}$ $\qquad$ set input $f(t) = 0$

ZSR: $y_f(t) = \mathcal{L}^{-1}\left\{\dfrac{P(s)}{Q(s)}F(s)\right\}$ $\qquad$ set ICs to 0

Use $Y(s) = H(s)F(s)$ to find the sys ODE

BIBO Stability of LTI Systems

$$H(s) = \frac{P(s)}{Q(s)} = \beta_n + \frac{b_{n-1}s^{n-1} + \cdots + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_0}$$

$\mathcal{H}$ is BIBO stable iff all poles of $H(s)$ have a -ve real part.

If poles of a LTI sys has a -ve real part, then $h(t) = \mathcal{L}^{-1}\{H(s)\}$ admits a FT $H(j\omega) = \mathcal{F}\{h(t)\}$. Moreover, $H(j\omega) = H(s)|_{s=j\omega}$.

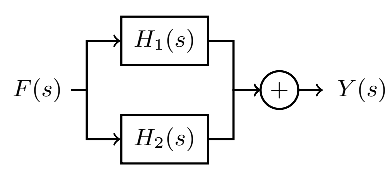
Block Diagrams

Cascading Combination

$F(s) \longrightarrow \boxed{H_1(s)} \longrightarrow \boxed{H_2(s)} \longrightarrow Y(s) = H_1(s)H_2(s)F(s)$

$H(s) = H_1(s)H_2(s)$

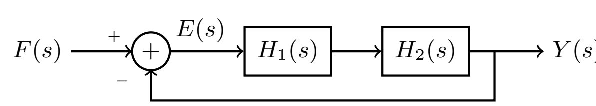
Sum Combination



$Y(s) = H(s)F(s)$

$H(s) = H_1(s) + H_2(s)$

Feedback Combination



$E(s) = F(s) - Y(s)$

$Y(s) = E(s)H_1(s)H_2(s) = [F(s) - Y(s)]H_1(s)H_2(s)$

$$Y(s) = \frac{H_1(s)H_2(s)}{1+H_1(s)H_2(s)}F(s)$$

FT Properties

Operation	Time domain	Freq domain
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(\omega) + a_2 F_2(\omega)$
Symmetry	$F(t)$	$2\pi f(-\omega)$
Scaling	$f(at)$	$\frac{1}{\|a\|} F\left(\frac{\omega}{a}\right)$
Time shift	$f(t - t_0)$	$F(\omega) e^{-j\omega t_0}$
Freq shift	$f(t) e^{j\omega_0 t}$	$F(\omega - \omega_0)$
Time convolution	$f_1(t) * f_2(t)$	$F_1(\omega) F_2(\omega)$
Freq convolution	$f_1(t) f_2(t)$	$\frac{1}{2\pi} F_1(\omega) * F_2(\omega)$
Time differentiation	$\frac{d^n f}{dt^n}$	$(j\omega)^n F(\omega)$
Time integration	$\int_{-\infty}^{t} f(x)\ dx$	$\frac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega)$
Periodic signal	$f(t) = f(t + T)$	$2\pi \sum_{n=-\infty}^{\infty} D_n \delta(\omega - n\omega_0)$

Symmetry/duality: $\mathcal{F}\{F(t)\} = 2\pi f(-\omega)$

Symmetry example: Find $f(t) = \mathcal{F}^{-1}\{F(\omega) * \delta(\omega - \omega_0)\}$ if $F(\omega) = \text{rect}\left(\frac{\omega}{d}\right)$.

1. $F(\omega) * \delta(\omega - \omega_0) = \int_{-\infty}^{\infty} F(\tau)\delta(\omega - \omega_0 - \tau)\ d\tau = F(\omega - \omega_0)$
2. Recall $F(t) = 2\pi f(-\omega) \implies \mathcal{F}\{d\text{sinc}\left(\frac{td}{2}\right)\} = 2\pi\text{rect}\left(-\frac{\omega}{d}\right) = 2\pi\text{rect}\left(\frac{\omega}{d}\right)$.
3. Also recall $\mathcal{F}^{-1}\{F(\omega - \omega_0)\} = f(t)e^{j\omega_0 t} \implies \mathcal{F}^{-1}\{\text{rect}\left(\frac{\omega - \omega_0}{d}\right)\} = \frac{d}{2\pi}\text{sinc}\left(\frac{td}{2}\right) e^{j\omega_0 t}$

. . . .

FT of Important Functions

	$f(t)$	$F(\omega)$		$f(t)$	$F(\omega)$
1	$\delta(t)$	1	10	$e^{j\omega_0}$	$2\pi\delta(\omega - \omega_0)$
2	1	$2\pi\delta(\omega)$	11	$\cos\omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
3	$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$	12	$\sin\omega_0 t$	$j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$
4	$\text{sgn}\ t$	$\frac{2}{j\omega}$	13	$\cos\omega_0 t\ u(t)$	$\frac{\pi}{2}[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)] + \frac{j\omega}{\omega_0^2 - \omega^2}$
5	$e^{-at}u(t),\ a > 0$	$\frac{1}{a + j\omega}$	14	$\sin\omega_0 t\ u(t)$	$\frac{\pi}{2j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)] + \frac{\omega_0}{\omega_0^2 - \omega^2}$
6	$e^{at}u(-t),\ a > 0$	$\frac{1}{a - j\omega}$	15	$e^{-at}\cos\omega_0 t\ u(t),\ a > 0$	$\frac{a + j\omega}{(a + j\omega)^2 + \omega_0^2}$
7	$te^{-at}u(t),\ a > 0$	$\frac{1}{(a + j\omega)^2}$	16	$e^{-at}\sin\omega_0 t\ u(t),\ a > 0$	$\frac{\omega_0}{(a + j\omega)^2 + \omega_0^2}$
8	$t^n e^{-at}u(t),\ a > 0$	$\frac{n!}{(a + j\omega)^{n+1}}$	17	$\text{rect}(t/d)$	$d\ \text{sinc}\frac{\omega d}{2}$
9	$e^{-a\|t\|},\ a > 0$	$\frac{2a}{a^2 + \omega^2}$			

$\mathcal{F}\{f(t)\cos\omega_0 t\} = \frac{1}{2}\left[F(\omega + \omega_0) + F(\omega - \omega_0))\right]$

$\mathcal{F}\{f(t)\cos^2\omega_0 t\} = \frac{1}{2}F(\omega) + \frac{1}{4}\left[F(\omega + 2\omega_0) + F(\omega - 2\omega_0)\right]$

LT Properties

Operation	Time domain	Laplace domain
Scaling	$f(at), \quad a \geq 0$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
Time convolution	$f_1(t) * f_2(t)$	$F_1(s)F_2(s)$
Time differentiation	$\frac{df}{dt}$	$sF(s) - f(0^-)$
	$\frac{d^2f}{dt^2}$	$s^2F(s) - sf(0^-) - f'(0^-)$
	$\frac{d^3f}{dt^3}$	$s^3F(s) - s^2f(0^-) - sf'(0^-) - f''(0^-)$
Time integration	$\int_{0^-}^{t} f(x)\ dx$	$\frac{1}{s}F(s)$
	$\int_{-\infty}^{t} f(x)\ dx$	$\frac{1}{s}F(s) + \frac{1}{s}\int_{-\infty}^{0^-} f(t)\ dt$
Time shift	$f(t-t_0)u(t-t_0)$	$F(s)e^{-st_0}, \quad t_0 \geq 0$
Frequency shift	$f(t)e^{s_0 t}$	$F(s - s_0)$
Frequency differentiation	$-tf(t)$	$\frac{dF(s)}{ds}$
Initial value thm	$f(0^+)$	$\lim_{s\to\infty} sF(s)$
Final value thm	$\lim_{t\to\infty} f(t)$	$\lim_{s\to 0} sF(s)$
		(poles of $sF(s)$ in LHP)
Scale & shift	$f(at-b)$	$\frac{e^{-bs/a}}{a}F\left(\frac{s}{a}\right)$

Unilateral LT of Important Functions

	$f(t)$	$F(s)$		$f(t)$	$F(s)$
1	$\delta(t)$	1	7	$t^n e^{\lambda t}u(t)$	$\frac{n!}{(s-\lambda)^{n+1}}$
2	$u(t)$	$\frac{1}{s}$	8	$\cos(bt)u(t)$	$\frac{s}{s^2+b^2}$
3	$tu(t)$	$\frac{1}{s^2}$	9	$\sin(bt)u(t)$	$\frac{b}{s^2+b^2}$
4	$t^n u(t)$	$\frac{n!}{s^{n+1}}$	10	$e^{-at}\cos(bt)u(t)$	$\frac{s+a}{(s+a)^2+b^2}$
5	$e^{\lambda t}u(t)$	$\frac{1}{s-\lambda}$	11	$e^{-at}\sin(bt)u(t)$	$\frac{b}{(s+a)^2+b^2}$
6	$te^{\lambda t}u(t)$	$\frac{1}{(s-\lambda)^2}$			