Multivariate Probabilities Formula Sheet

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Discrete Pairs of Random Variables

Joint pmf: $p(x, y) = P(X = x \text{ and } Y = y)$

$$0 \leq p(x, y) \leq 1 \qquad \sum_{(x,y)}\sum p(x, y) = 1$$

$$P((X, Y) \in A) = \sum_{(x,y) \in A}\sum p(x, y)$$

Joint CDF: $F(x, y) = P(X \leq x \text{ and } Y \leq y) = \sum_{x_i \leq x} \sum_{y_i \leq y} p(x_i, y_j)$

Marginal PMF: $p_X(x) = \sum_j p(x_i, y_j) \qquad p_Y(y) = \sum_i p(x_i, y_j)$

Expectation: $\mathbb{E}[g(X, Y)] = \sum_{(x,y) \in A}\sum g(x, y)p(x, y)$

Conditional expectation: $\mathbb{E}[X \mid A] = \sum_x xP(X = x \mid A)$

$$\mathbb{E}[X, Y] = xy \cdot p(x, y) \qquad \mathbb{E}[X] = x \cdot p(x, y) \qquad \mathbb{E}[Y] = y \cdot p(x, y)$$

Independent RVs: $p(x_1, x_2, \ldots, x_n) = p_{X_1}(x_1)p_{X_2}(x_2) \cdots p_{X_n}(x_n)$

$$\mathbb{E}[XY] = \mathbb{E}[X]\mathbb{E}[Y]$$

$P(A \cap B) = P(A \mid B)P(B) = P(B \mid A)P(A)$

$p_{Y|X}(y \mid x) = P(Y = y \mid X = x) = \dfrac{P(X = x \text{ and } Y = y)}{P_X(x)}$

Continuous Pairs of Random Variables

Joint PDF: $F_{X,Y}(x, y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f_{X,Y}(u, v)\ du\ dv$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{X,Y}(u, v)\ du\ dv = 1$$

$$P[x_0 < x < x_0 + dx, y_0 < Y < y_0 + dy] = f_{X,Y}(x_0, y_0)\ dx\ dy$$

$$f_{X,Y}(x, y) = \frac{\partial^2 F_{X,Y}(x, y)}{\partial x \partial y}$$

$$P[A] = \iint_{(x,y) \in A} f_{X,Y}(x, y)\ dx\ dy$$

Marginal PDF: $f_X(x, y) = \int_{-\infty}^{\infty} f_{X,Y}(x, y)\ dy$

$$f_Y(x, y) = \int_{-\infty}^{\infty} f_{X,Y}(x, y)\ dx$$

Expectation: $\mathbb{E}[g(x, y)] = \iint_{(x,y) \in A} g(x, y)f_{X,Y}(x, y)\ dx\ dy$