Python 3 Bits & Bytes, File Compression

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1 Introduction to File Compression

1.1 Topics Covered

(i) Data as bits (ii) Compression (iii) Huffman coding

1.2 Bit

- Bit is basic unit of mem in computer
- 8 bits = 1 byte
- 4 bits = 1 nibble
- 2 bits = 1 crumb

1.3 Characters in Text Files

- ASCII table defs binary rep of 256 (2^8) diff chars
- Each char has 8 bit representation: 1 byte
- ASCII table is w/in Unicode table

1.4 Always Need All Those Bits?

- If 1 char = 1 byte, text file w/ 1000 chars contains ~1kB of data
- What if text file only contains chars 'a' & 'b'?
  - Woudn't need full byte → just use single bit: 'a' = 0, 'b' = 1
  - Using this approach, 1000 a's & b's would take up 1/8 space of normal ASCII rep

2 Data Compression

2.1 Compression Intro

- Basic goal of compression: rep file using fewer bits, even if we have to store contents in unconventional format
- Pros: use less mem to store file, transmit files faster

2.2 Another Example

2.2.1 Method 1

- What if file has chars 'a', b', 'n' (ex: banana)?
- Can use 'a' = 00, 'b' = 01, 'c' = 10 → banana = 010010001000
- This gives compression rate of 1/4 that of plaintext file

2.2.2 Better Compression Choice?

- Is there better way to compress text file w/ only 3 chars? YES!
- Assoc most freq character w/ 0, and remaining w/ 10 & 11
- Compression rate:
  - at least 1/3 of chars go from 8 bits to 1 bit
  - Remaining chars go from 8 bits to 2 bits

2.3 Calculate Compression Rate

- n_i = # times 'i' occurs (where $i = a, b, c$)
- $n = n_a + n_b + n_n$
- Suppose 'a' is most freq. Then $n_a \geq n/3$
- Amount of bits used in compressed string is:

$$\begin{aligned} bits &= 1 * n_a + 2 * n_b + 3 * n_n \\ &= 2 * n - n_a \\ &\leq 2 * n - n/3 \\ &\leq 5/3n \end{aligned}$$

- # bytes is $(5/3n)/8 = 0.2083$ or less
- Better than 0.25n we got b4 when all 3 chars were 2 bit seq

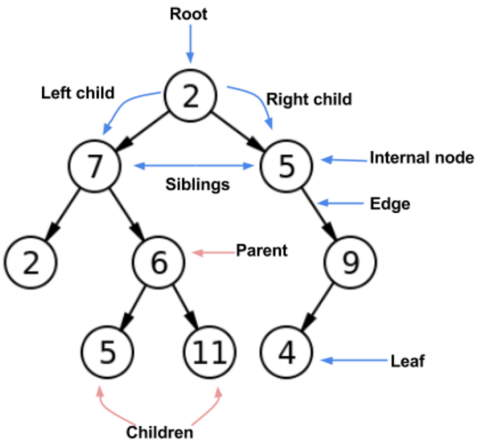
2.4 Decoding

- Can we decode compressed file when some chars encoded w/ 1 bit, others w/ 2 bits?
- Decode = 10010011
- 'a' = 0, 'b' = 10, 'n' = 11

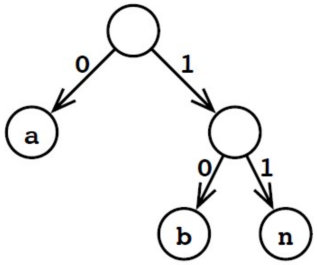
3 Decoding Tree

3.1 Intro to Decoding Tree

- If no bit seq is beginning of another in encoding, we can build decoding tree
- 0/1 labels on edges of root-to-leaf path = encoding of char in given leaf

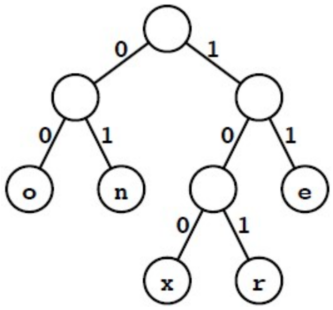


- All chars are leaf nodes, all edges are 0/1
- If node not leaf, then it's an internal node
- Binary trees have at MOST 2 children



3.2 How to Use Decoding Tree

- Decode bit seq using bits to traverse given tree
  1. Start at root, follow 0/1 edge according to next bit in seq
  2. Output char at leaf when you reach one
  3. Return to root & repeat for next branch
- 001001101 → 00 (o) + 100 (x) + 11 (e) + 01 (n) = oxen

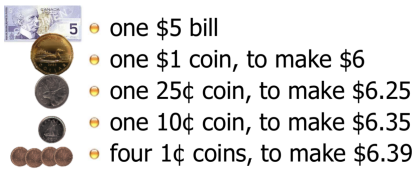


3.3 Build a Decoding Tree

- The key is picking encoding for each char
- **Req:** no bit seq for any char is the beginning (prefix) of another bit seq → this type of encoding scheme called prefix code
- **Desire:** chars that occur more freq should have shorter bit seqs
- **Optimization problem:** construct decoding tree to minimize total # of bits used to compress file
- This can be achieved using Huffman Trees ~ trees constructed according to simple **greedy procedure**

3.4 Greedy Algorithms

- At each step:
  - Take best step we can get right now, w/o regard to eventual optimization
  - Hope that by choosing local optimum at each step, you'll end up at global optimum
- Ex: count $6.39, using fewest bills & coins
  - Greedy algorithm: at each step, take largest bill/coin that does not overshoot



4 Huffman Coding

4.1 Building a Huffman Tree

1. Do freq count of all chars in file (include count of 1 for EOF sentinel)
   (a) Ex: freq['a'] = 10, freq['w'] = 2, freq[EOF] = 1
   (b) Ultimately, keys will be bytes, not chars
   (c) Total freq count of tree is sum of freqs of its leaves
2. Initially, each char is single node in trivial Huffman tree
3. While there is more than 1 tree, merge the 2 w/ the smallest freq counts
   (a) Make each tree a child of a new root node
   (b) Doesn't matter which tree is left/right child
   (c) # on this new tree is total freq count of all leaves
   (d) Repeat: pick 2 trees w/ lowest total freq count, & merge (in case of tie, doesn't matter which one you pick)
4. Merge trees T_1 & T_2 means creating a new root node & setting T_1 & T_2 as its children

4.2 Summary: Compress the File

- For each char, det its 0/1 compression encoding by looking at the root-to-leaf path
- Output the seq of 0/1 bits obtained by replacing the char w/ its compressed bits
- Don't forget the final seq for EOF sentinel

4.3 Summary: Decompress the file

- Starting from root, traverse Huffman tree. Each bit from input seq dictates when to go L/R
- When you reach a leaf, output the char, return to root, continue traversing tree according to next bit(s) in seq
- Quit when you reach the EOF leaf

4.4 Why Include an EOF?

- B/c last byte of compressed file might not be "complete" (ex: 35 bits in compression seq: 4 bytes & 3 bits, so EOF pads w/ 0); need multiple of 8 bits to transmit file
- ∴, decoding EOF tells us when to stop

4.5 Considerations

- To decompress a file using this approach, need to know struc of Huffman tree used to compress file in 1st place
- When we send compressed file, also need to send rep of Huffman tree used

4.6 Final Notes

- Huffman compression exploits freqs of chars → fairly simple compression scheme, but results in reduced file sizes if file contains **subset** of the 2^8 diff bytes (chars); no point in using Huffman tree if there are 256 diff chars in file
- Huffman compression tends to work best on plain text files & bitmap images w/ small range of colours
- Other compression schemes exploit other patterns, & often target spec file types (pics, text, etc.). Some compressions allow for some data loss (ex: w/ .jpeg files)
- No compression can make every file smaller